ICFO

Victoria J Wright

Introduction to contextuality

Quantum theory inherently probabilistic

Upsetting:

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Could it be deterministic but we just don’t know everything?

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State

$|0\rangle$

Quantum theory inherently probabilistic

Upsetting:

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State

$|0\rangle$

Measurement

$\{|+\rangle\langle+|, |-\rangle\langle-|\}$

$1/2 \quad 1/2$

Quantum theory inherently probabilistic

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State	Measurement		Actually	
$\lvert 0\rangle$	$\{\lvert +\rangle\langle +\rvert, \lvert -\rangle\langle -\rvert\}$		λ_+	λ_-
	$1/2$	$1/2$	$1/2$	$1/2$

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Yes!

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Yes!

But…then it also must be (measurement) contextual

Observables that are (statistically) indistinguishable to us, are in fact different!

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KOCHEN-SPECKER THEOREM

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CONTEXTS OF $Z \otimes I$

① $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

+1 +1 -1 -1 ZI

② $\{|0+\rangle, |0-\rangle, |1+\rangle, |1-\rangle\}$

+1 +1 -1 -1 ZI

Two different contexts for the same measurement

- Context: which other observables I simultaneously measure

OR which experimental procedure I use

Noncontextuality:

① ZI = ② ZI

fundamentally same observable

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+1	-1	+1	-1	IZ
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+1	+1	-1	-1	ZI
+1	-1	+1	-1	IX
+1	-1	-1	+1	ZX

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DETERMINISTIC NONCONTEXTUAL ONTOLOGICAL MODELS

Ontic states Λ

Epistemic states μ_ρ

DETERMINISTIC NONCONTEXTUAL ONTOLOGICAL MODELS

Ontic states

Λ

[SPEC] $\lambda(A) \in \sigma(A)$

[FUNC] $\lambda(f(A)) = f(\lambda(A))$

Epistemic states

μ_ρ

$$P(A=a \mid \rho) = \int_{\lambda \text{ s.t. } \lambda(A)=a} \mu_\rho(\lambda)\, d\lambda$$

probability measure on Λ

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E.g. $\lambda(A^2) = [\lambda(A)]^2$

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ZI IZ ZZ

IX XI XX

ZX XZ YY

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ZI	IZ	ZZ	I
IX	XI	XX	I
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I	I	-I	

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$\Downarrow$

$\lambda(AB) = \lambda(A)\lambda(B)$ if $[A,B]=0$

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$$\lambda(ZI)\lambda(IZ)\lambda(ZZ)\lambda(IX)\lambda(XI)\lambda(XX)\lambda(ZX)\lambda(XZ)\lambda(YY)$$

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IX	XI	XX	I
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I	I	-I	

$$\underbrace{\lambda(ZI)\lambda(IZ)\lambda(ZZ)}_{\lambda(I)}\underbrace{\lambda(IX)\lambda(XI)\lambda(XX)}_{\lambda(I)}\underbrace{\lambda(ZX)\lambda(XZ)\lambda(YY)}_{\lambda(I)} = 1$$

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Epistemic states

$$P(A=a \mid \rho) = \int_{\lambda \text{ s.t. } \lambda(A)=} \Lambda$$

ZI	IZ	ZZ	I
IX	XI	XX	I
ZX	XZ	YY	I
I	I	-I	

$$\underbrace{\lambda(ZI)\lambda(IZ)\lambda(ZZ)}_{\lambda(I)}\underbrace{\lambda(IX)\lambda(XI)\lambda(XX)}_{\lambda(I)}\underbrace{\lambda(ZX)\lambda(XZ)\lambda(YY)}_{\lambda(I)} = 1$$

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No deterministic noncontextual ontic states

$\Rightarrow$

KOCHEN-SPECKER THEOREM

there is no deterministic noncontextual ontological model for QT *

ontic states + epistemic states

Kochen, Specker (1967). "The problem of hidden variables in quantum mechanics". *Journal of Mathematics and Mechanics*. **17** (1): 59–87

* in dimensions ≥ 3

No deterministic noncontextual ontic states

also follows from Gleason's theorem [Exercise]

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Leifer, arXiv:1409.1570

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$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

+1	+1	-1	-1	ZI
+1	-1	+1	-1	IZ
+1	-1	-1	+1	ZZ

$\{|0+\rangle, |0-\rangle, |1+\rangle, |1-\rangle\}$

+1	+1	-1	-1	ZI
+1	-1	+1	-1	IX
+1	-1	-1	+1	ZX

$\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$

+1	-1	+1	-1	IX
+1	+1	-1	-1	XI
+1	-1	-1	+1	XX

$\{|+0\rangle, |-0\rangle, |+1\rangle, |-1\rangle\}$

+1	+1	-1	-1	IZ
+1	-1	+1	-1	XI
+1	-1	-1	+1	XZ

$\{|\alpha_+\rangle, |\alpha_-\rangle, |\beta_+\rangle, |\beta_-\rangle\}$

+1	+1	-1	-1	ZX
+1	-1	+1	-1	XZ
+1	-1	-1	+1	YY

$\{|\Phi_+\rangle, |\Phi_-\rangle, |\Psi_+\rangle, |\Psi_-\rangle\}$

+1	+1	-1	-1	ZZ
+1	-1	+1	-1	XX
-1	+1	+1	-1	YY

ZI	IZ	ZZ
IX	XI	XX
ZX	XZ	YY

$|\Phi_\pm\rangle = |00\rangle \pm |11\rangle$

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+1	-1	+1	-1	IX
+1	-1	-1	+1	ZX

$\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$

+1	-1	+1	-1	IX
+1	+1	-1	-1	XI
+1	-1	-1	+1	XX

$\{|+0\rangle, |-0\rangle, |+1\rangle, |-1\rangle\}$

+1	+1	-1	-1	IZ
+1	-1	+1	-1	XI
+1	-1	-1	+1	XZ

ZI	IZ	ZZ
IX	XI	XX
ZX	XZ	YY

$\{|\alpha_+\rangle, |\alpha_-\rangle, |\beta_+\rangle, |\beta_-\rangle\}$

+1	+1	-1	-1	ZX
+1	-1	+1	-1	XZ
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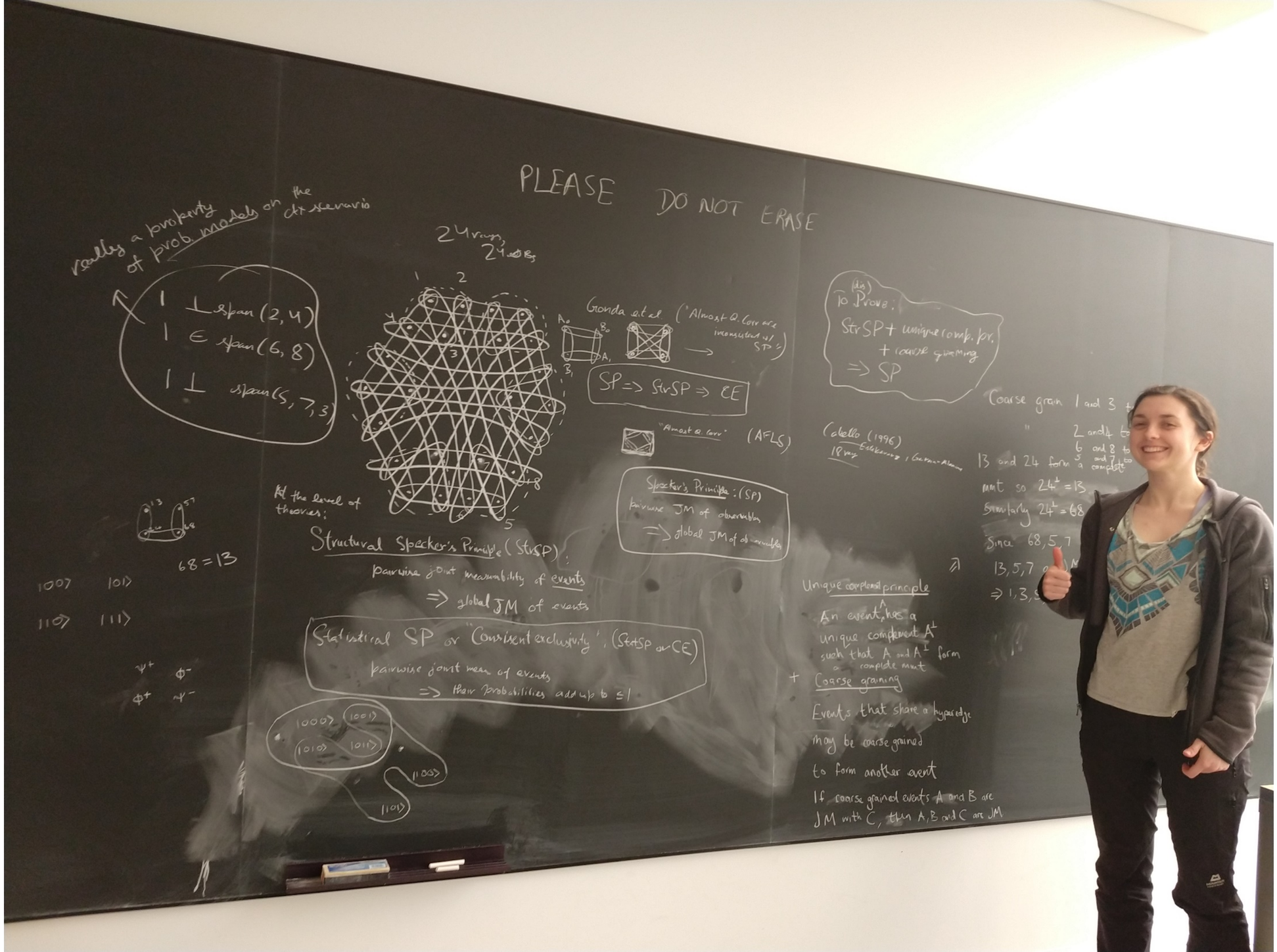
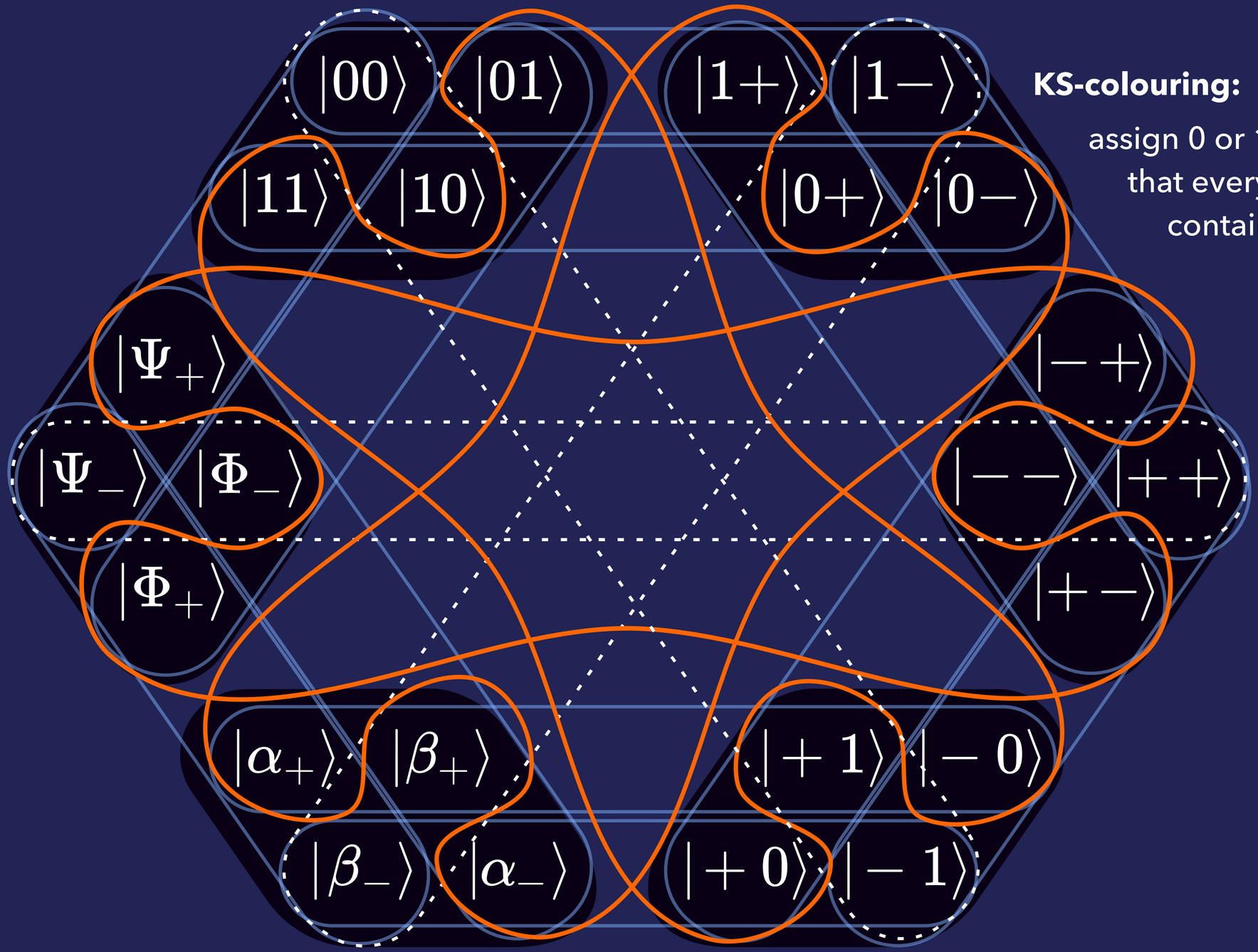
$\{|\alpha_+\rangle, |\Phi_+\rangle, |\alpha_-\rangle, |\Psi_-\rangle\}$


PLEASE DO NOT ERASE
really a property of prob. models on the ctx scenario
24 rays, 24 ...
Gonda et al. ("Almost Q. Corr are inconsistent w/ SP")
SP => StrSP => CE
"Almost a. corr"
(AFLS)
Cabello (1996)
18 ray
Eskelkeng, Gerhard-Albert
To (this) Prove:
StrSP + unique comp. pr.
+ coarse graining
=> SP
At the level of theories:
Structural Specker's Principle (StrSP):
pairwise joint measurability of events
=> global JM of events
Specker's Principle: (SP)
pairwise JM of observables
=> global JM of observables
Statistical SP or "Consistent exclusivity", (StatSP or CE)
pairwise joint meas of events
=> their probabilities add up to ≤ 1
Unique complement principle
An event A has a unique complement A⊥ such that A and A⊥ form a complete meas
+ Coarse graining
Events that share a hyperedge may be coarse grained to form another event
If coarse grained events A and B are JM with C, then A, B and C are JM
68 = 13
|00⟩ |01⟩
|10⟩ |11⟩
ψ+ φ−
φ+ ψ−
|000⟩ |001⟩
|010⟩ |011⟩
|100⟩
|101⟩
13 and 24 form a complete meas
so 24⊥ = 13
Similarly 24⊥ = 68
Since 68, 5, 7
13, 5, 7
⇒ 1, 3,



KS-colouring:

assign 0 or 1 to each ray such that every hyperedge contains exactly one 1

$|\Phi_\pm\rangle = |00\rangle \pm |11\rangle$
$|\Psi_\pm\rangle = |01\rangle \pm |10\rangle$
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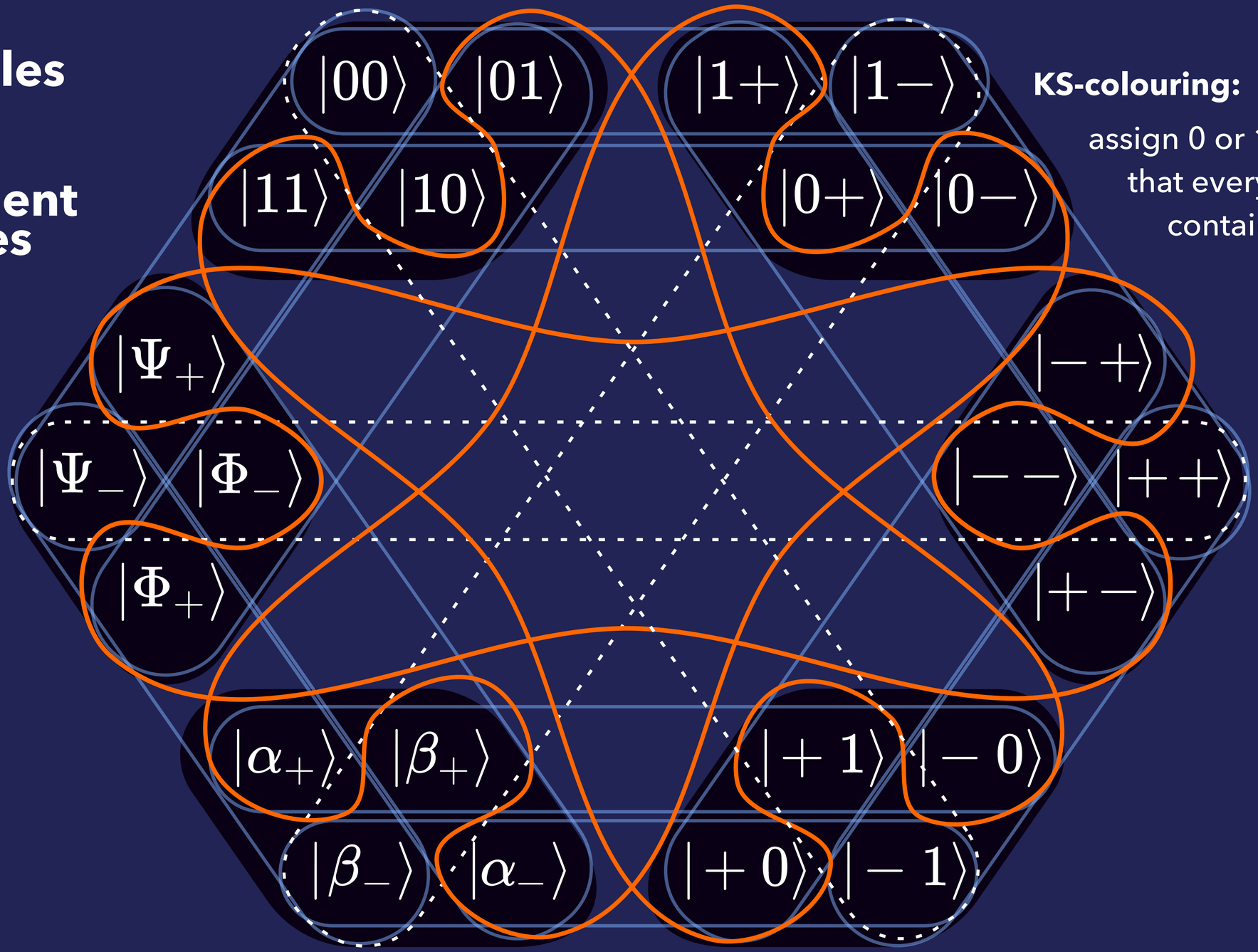
Observables vs measurement outcomes



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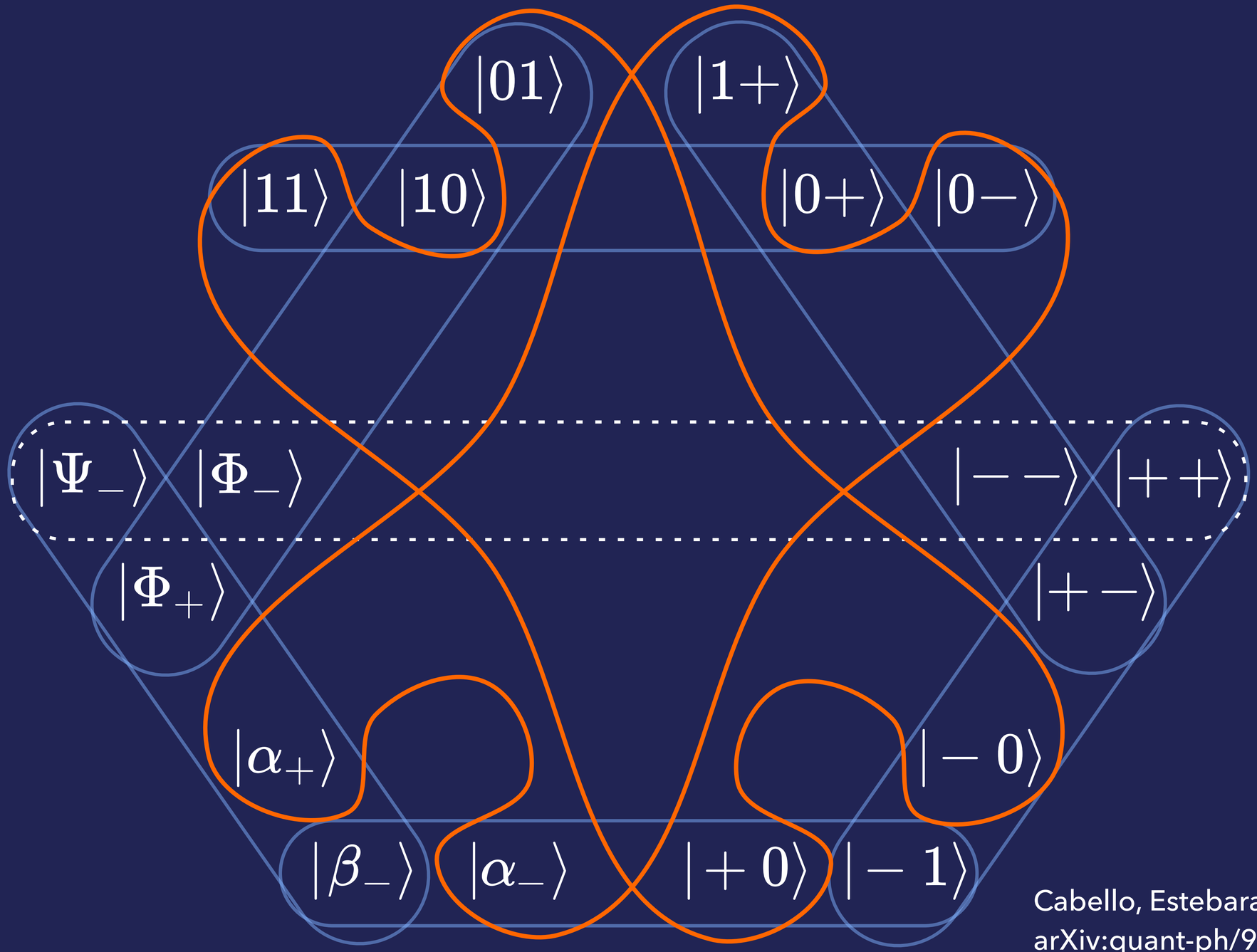

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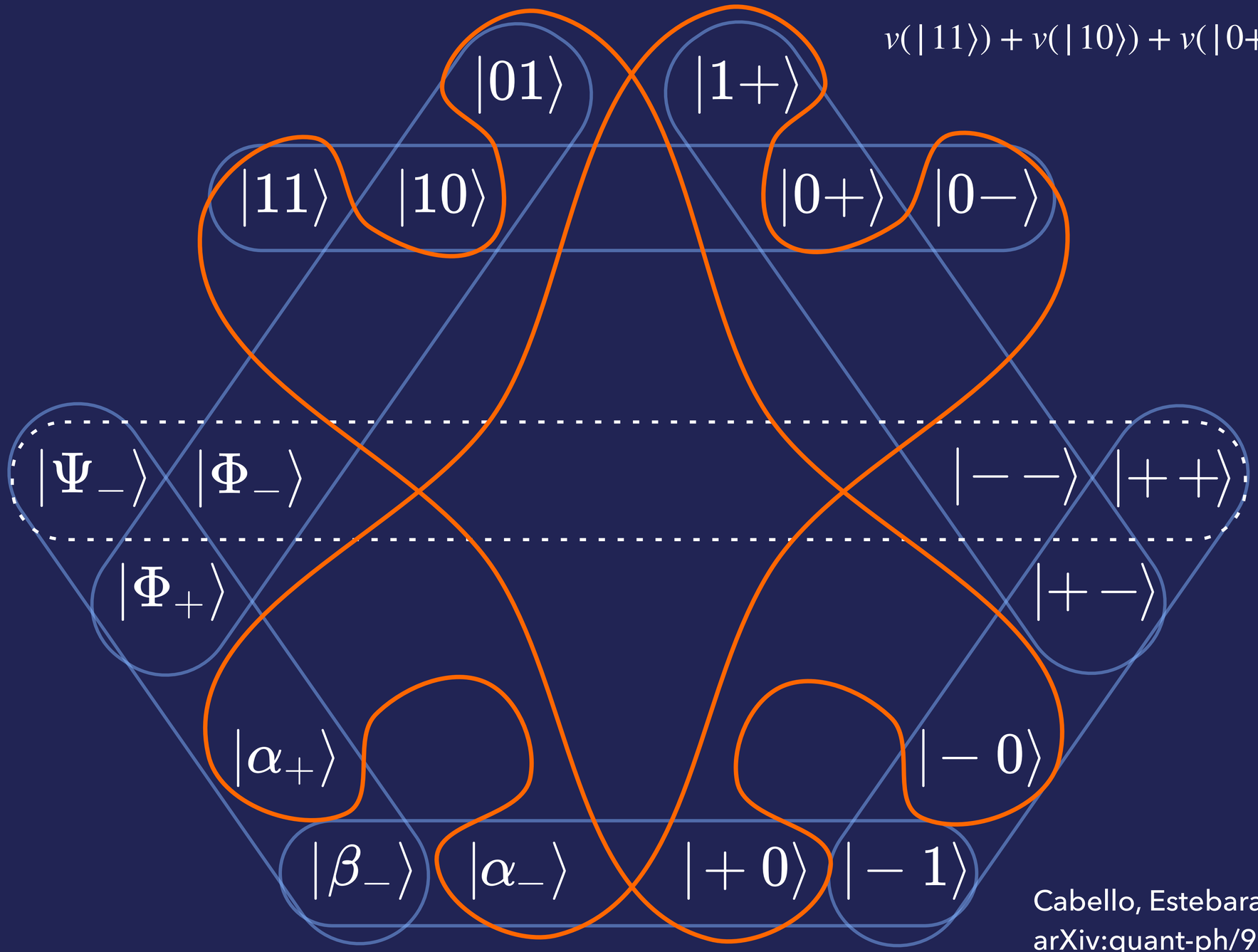
Cabello, Estebaranz, García-Alcaine
arXiv:quant-ph/9706009

$$v(|11\rangle) + v(|10\rangle) + v(|0+\rangle) + v(|0-\rangle) = 1$$



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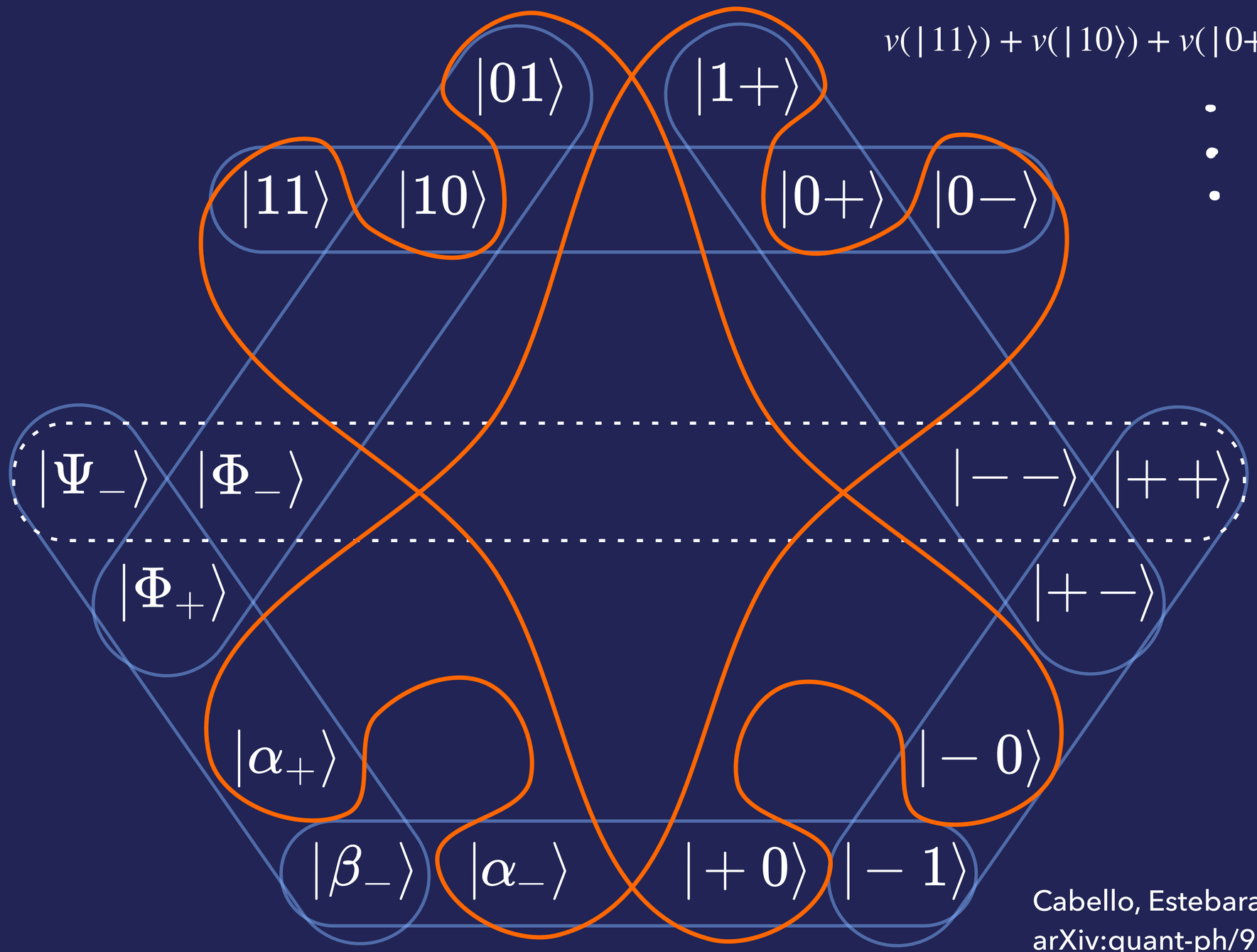
arXiv:quant-ph/9706009

$$v(|11\rangle) + v(|10\rangle) + v(|0+\rangle) + v(|0-\rangle) = 1$$



$$|\Phi_\pm\rangle = |00\rangle \pm |11\rangle$$
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Cabello, Estebaranz, García-Alcaine
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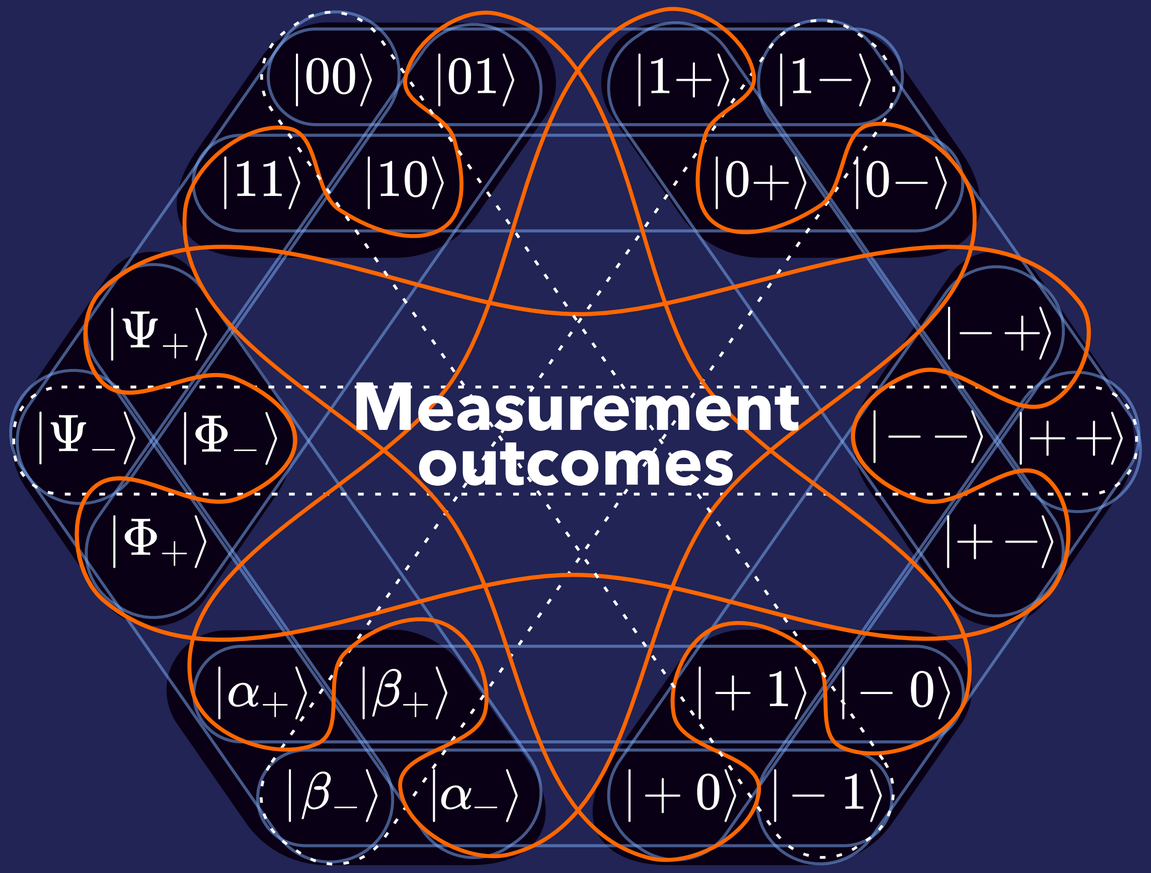

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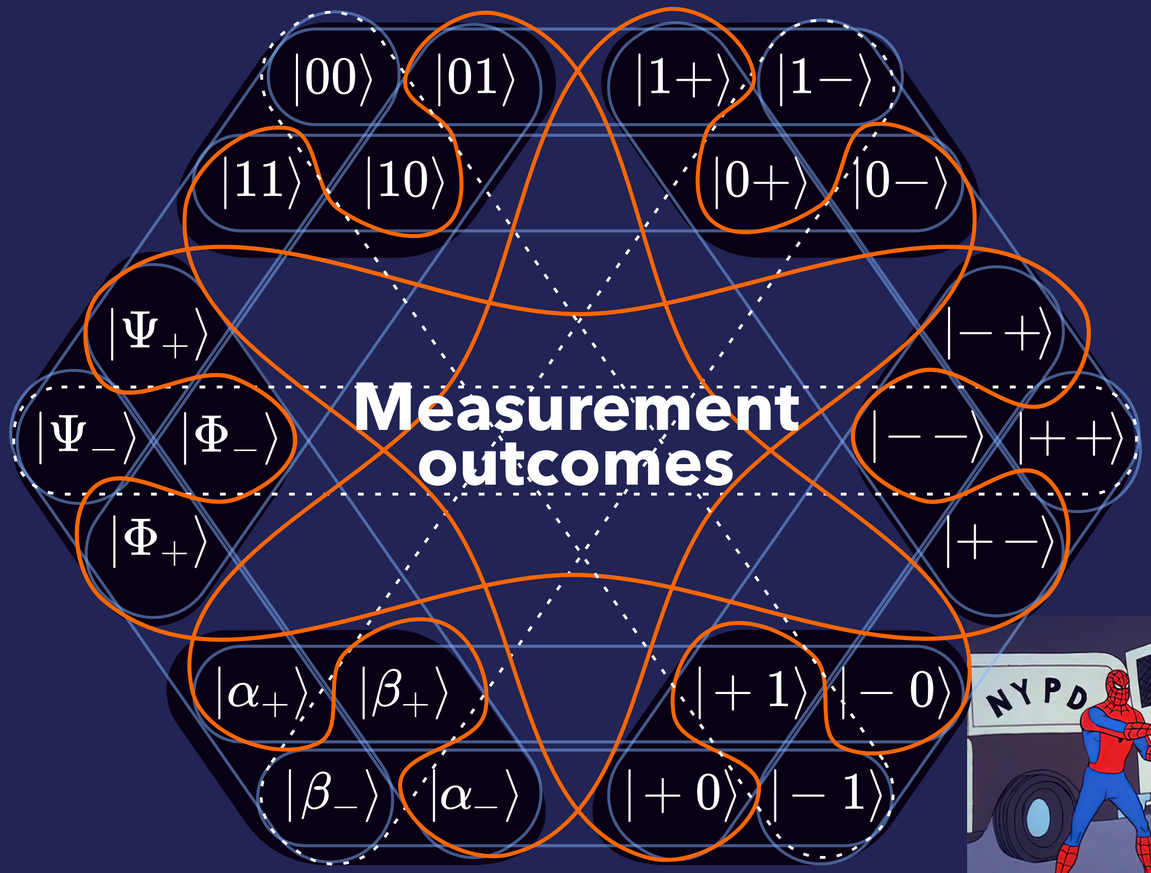
State independent proofs of KS theorem

No deterministic noncontextual ontic states

Observables

$$Z \otimes I \quad I \otimes Z \quad Z \otimes Z$$
$$I \otimes X \quad X \otimes I \quad X \otimes X$$
$$Z \otimes X \quad X \otimes Z \quad Y \otimes Y$$



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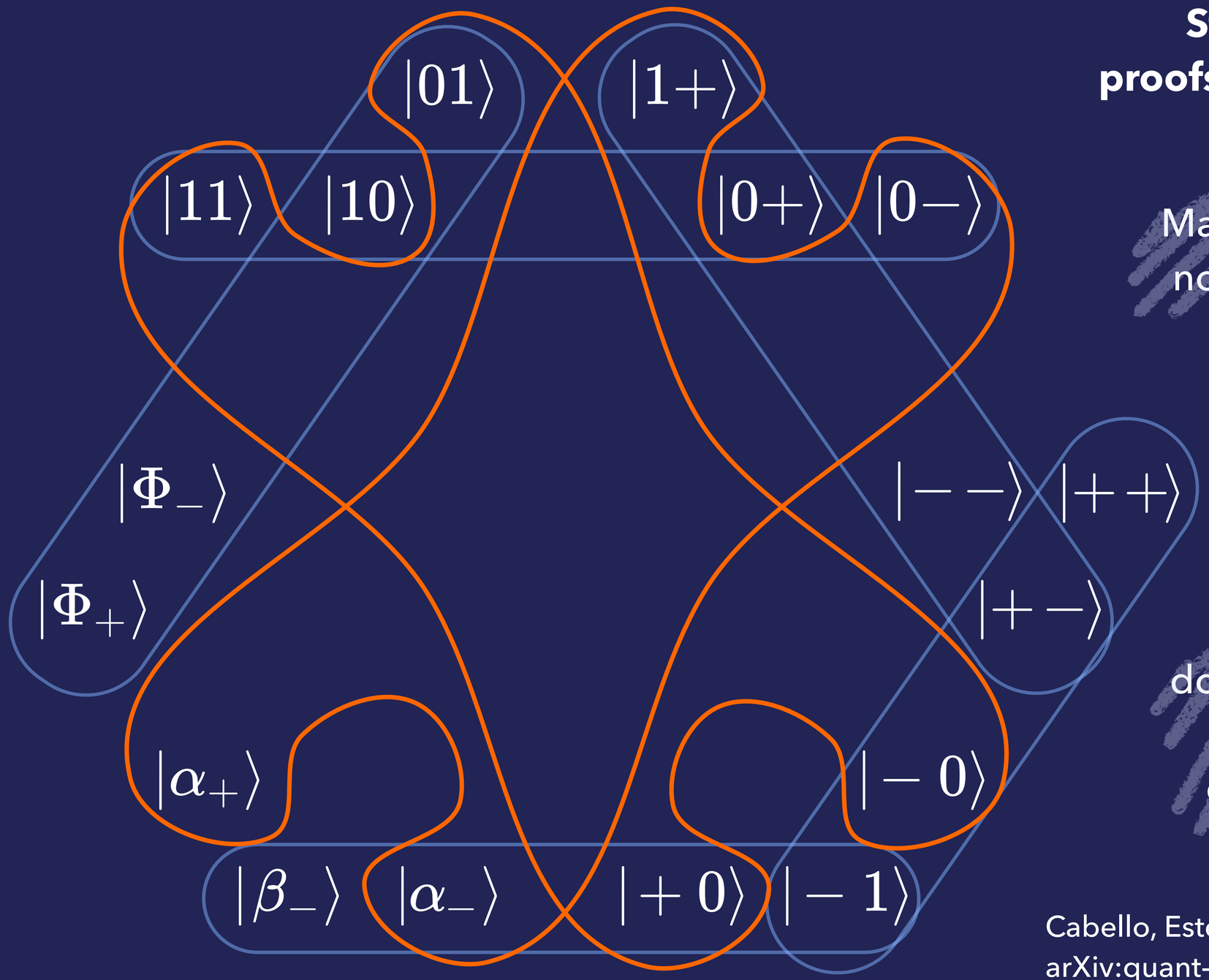

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BUT

do not mix together to reproduce quantum statistics

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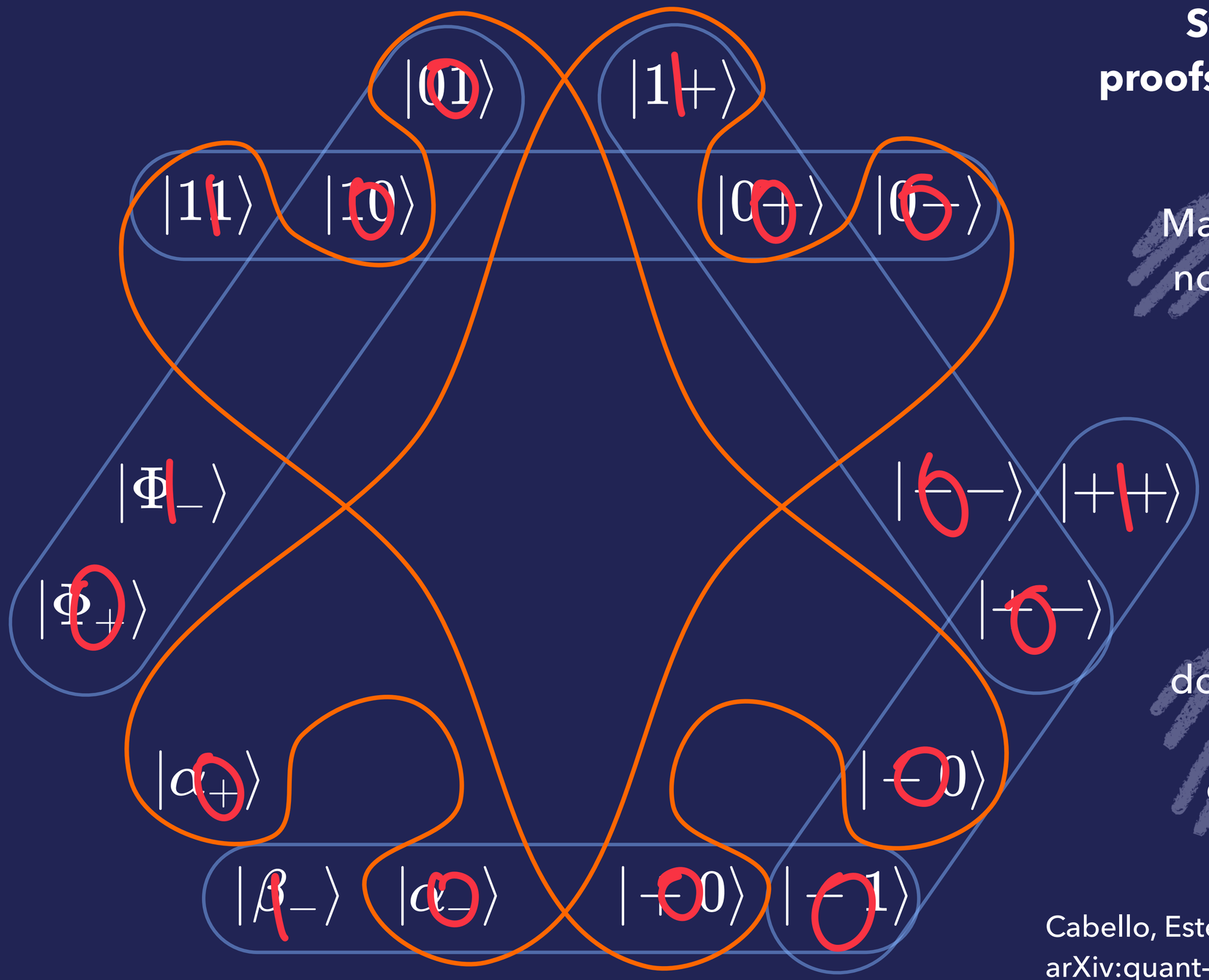

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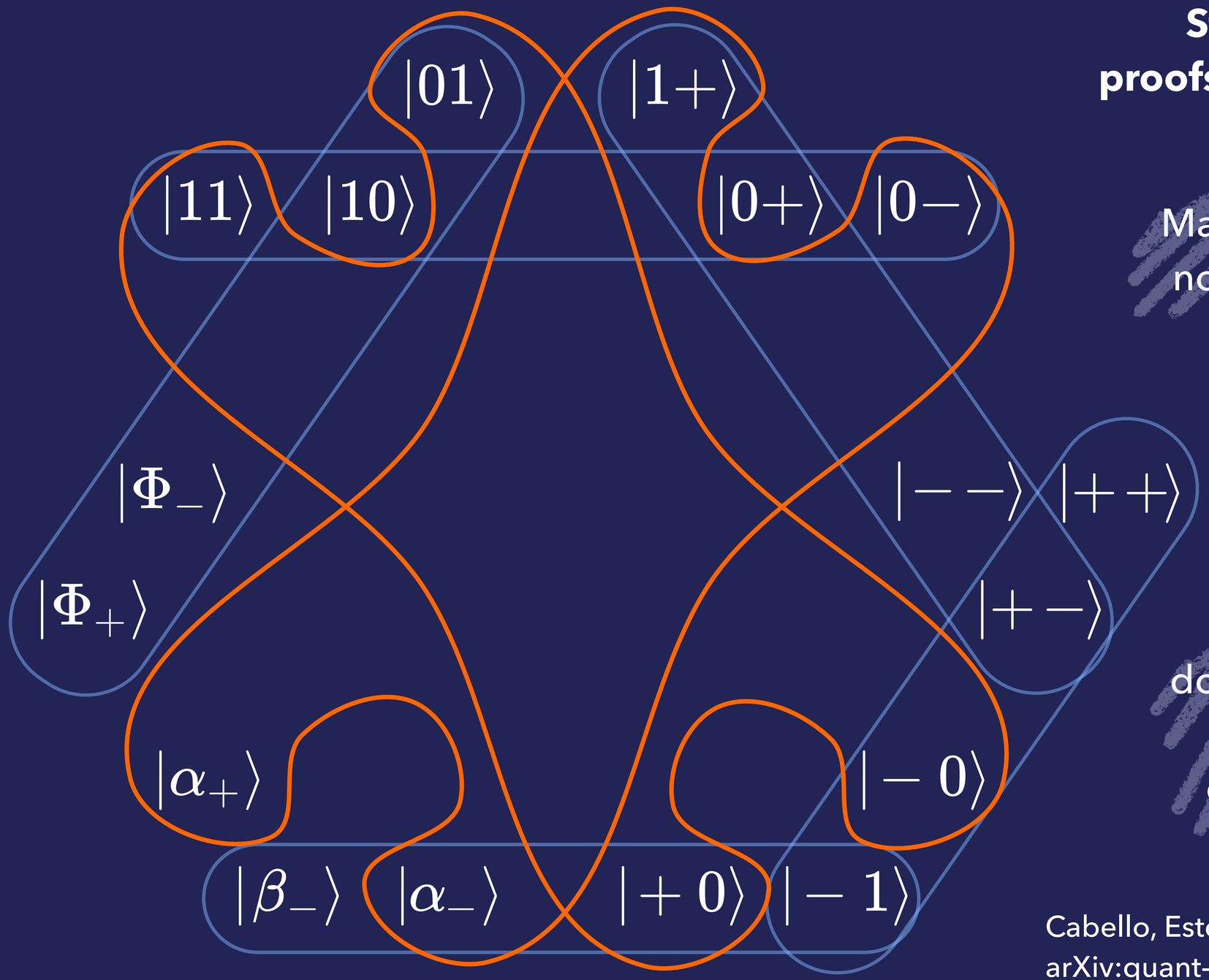

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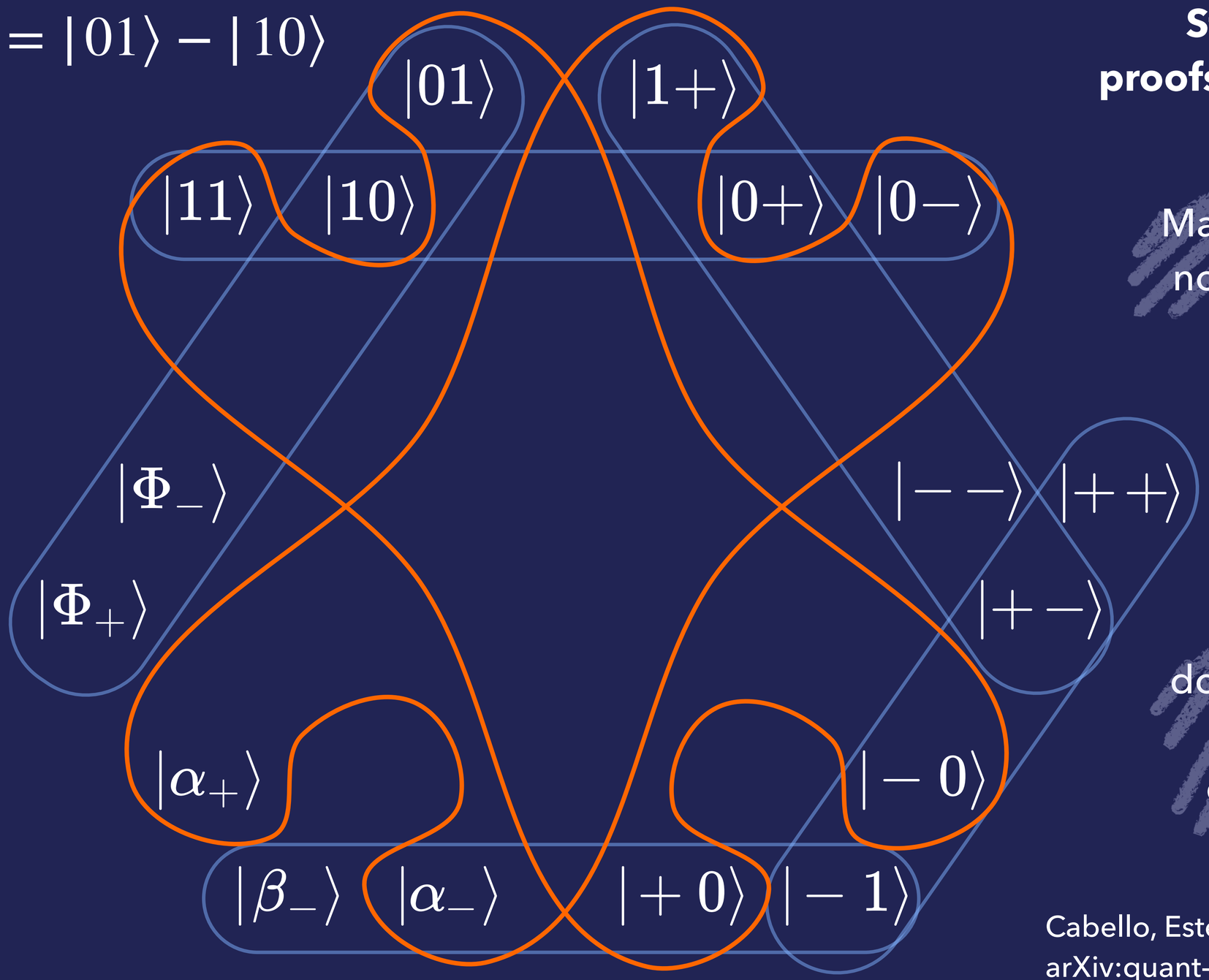
Cabello, Estebaranz, García-Alcaine
arXiv:quant-ph/9706009

State: $|\Psi_-\rangle = |01\rangle - |10\rangle$

State dependent proofs of KS theorem



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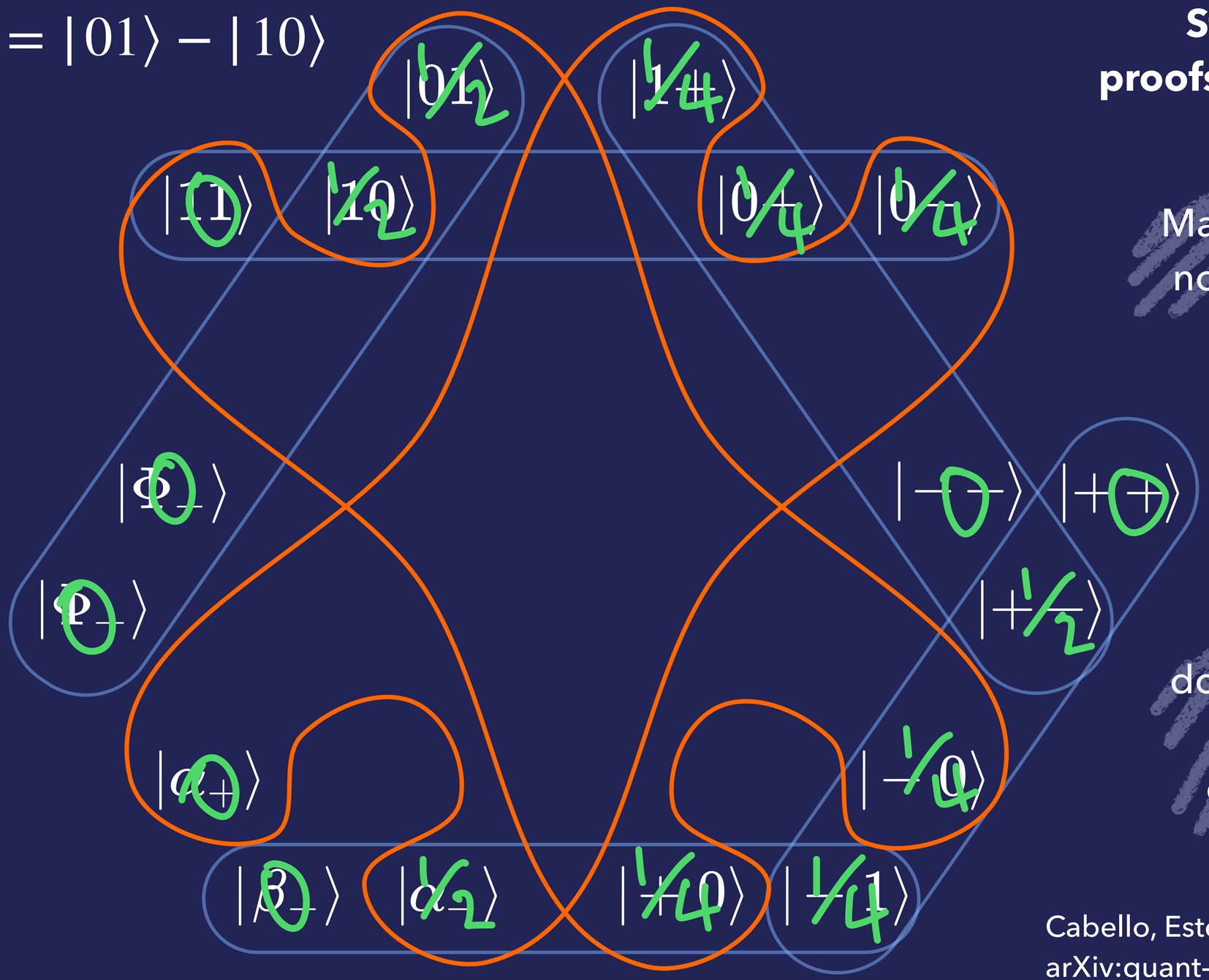

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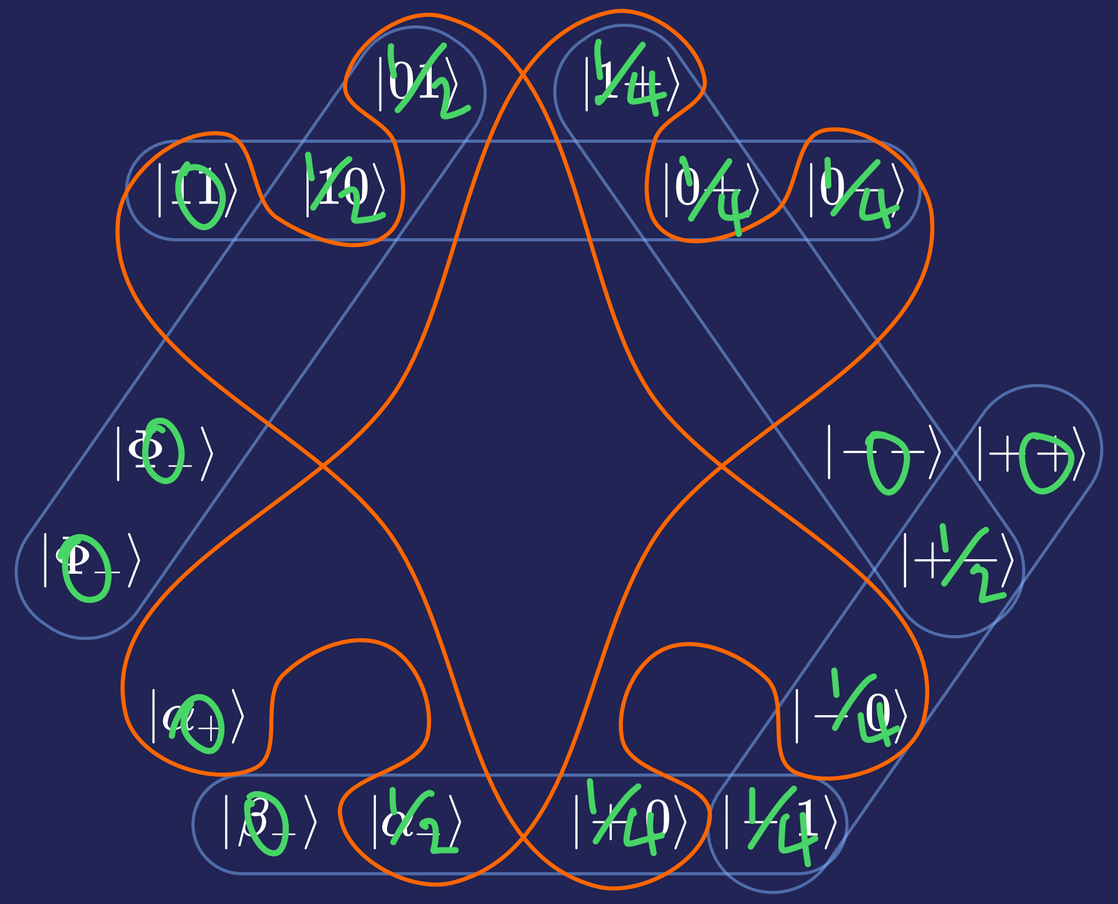

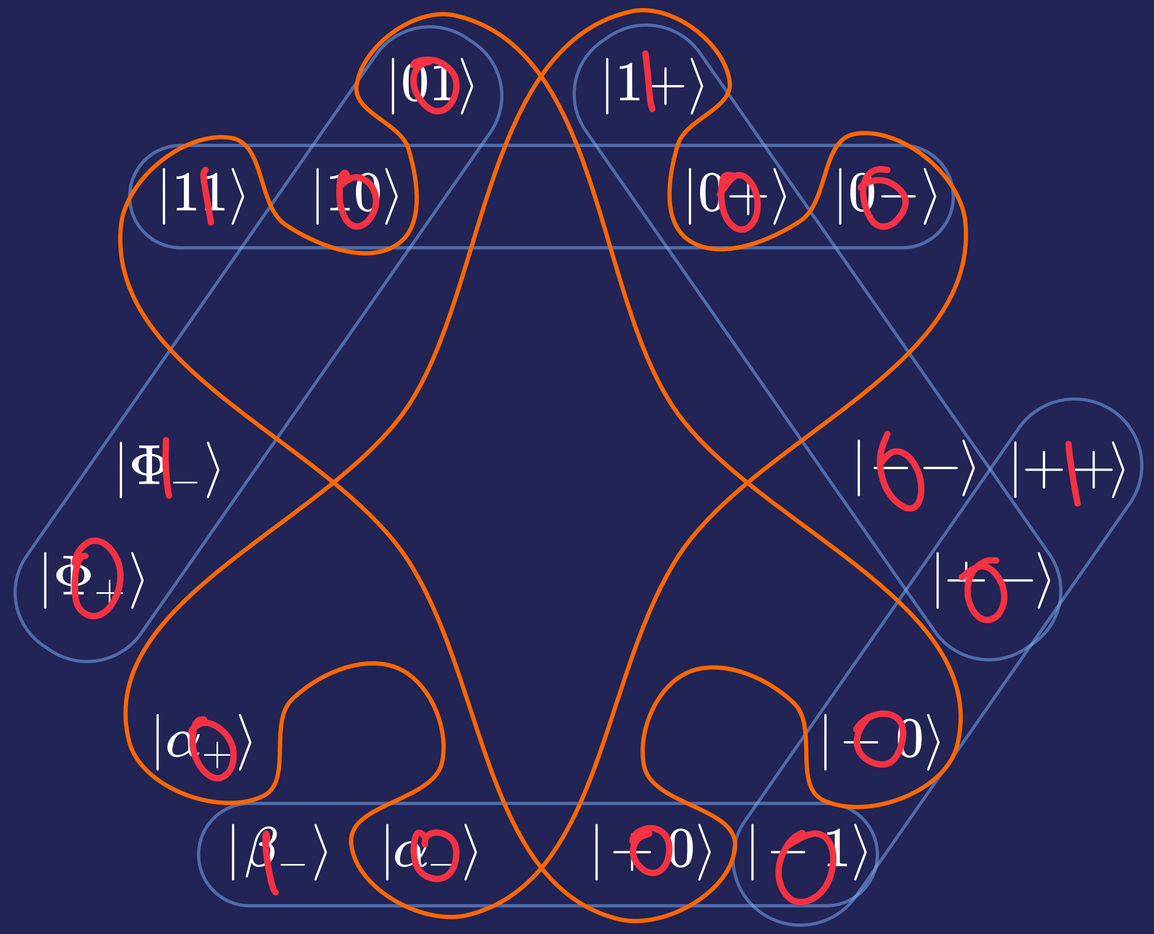

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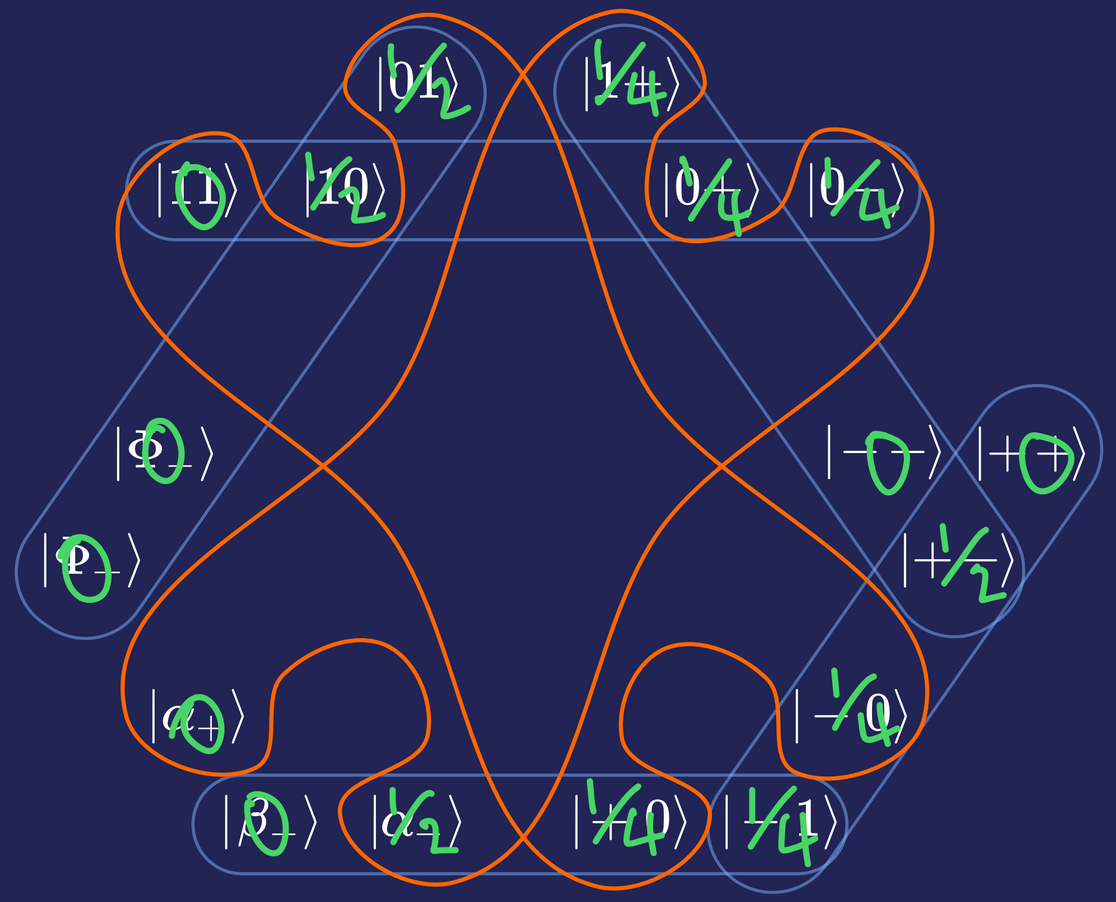
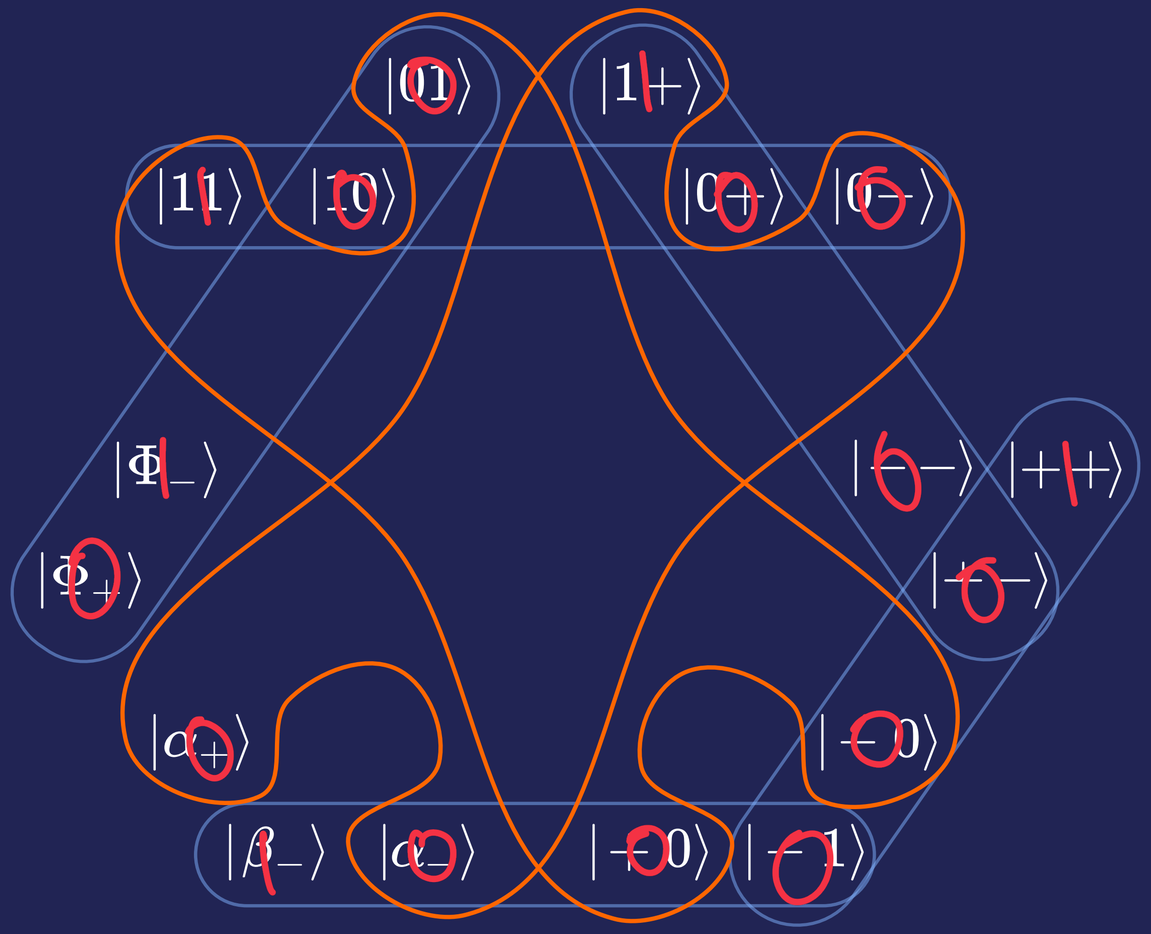
Exercise! Hint: think about the zeroes (you don't have to write out all the colourings)

Cabello, Estebaranz, García-Alcaine
arXiv:quant-ph/9706009

State: $|\Psi_-\rangle = |01\rangle - |10\rangle$

Also, noncontextuality inequalities e.g. Bell inequalities
Acín, Fritz, Leverrier, Sainz arXiv:1212.4084





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Kochen-Specker

DETERMINISM + MEASUREMENT NONCONTEXTUALITY

Difference between quantum and classical physics

Related to other forms of “nonclassicality”

Nonlocality:

- noncontextuality motivated via locality
- Bell inequality violations are state dependent proofs

Requires incompatibility

Spekkens

~~DETERMINISM~~ + MEASUREMENT NONCONTEXTUALITY

Preparation noncontextuality (+ transformation noncontextuality)

Spekkens, arXiv:quant-ph/0406166

Spekkens

~~DETERMINISM~~ + MEASUREMENT NONCONTEXTUALITY

Preparation noncontextuality (+ transformation noncontextuality)

Spekkens, arXiv:quant-ph/0406166

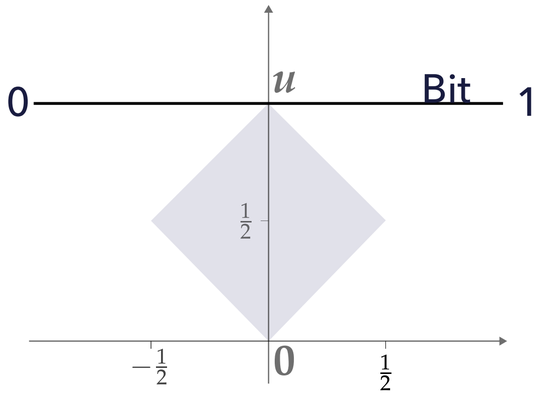
DIFFERENCES WITH KOCHEN-SPECKER

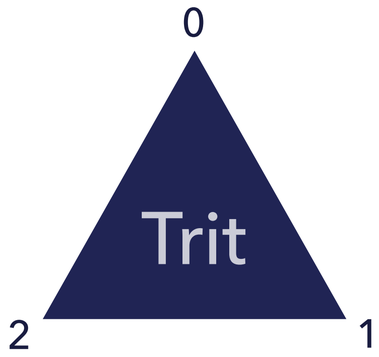
POVMs not just PVMs

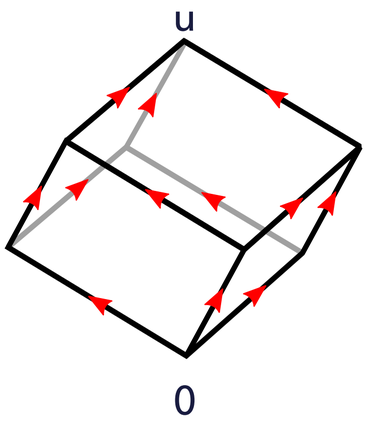
Dimension 2

Makes sense for general operational theories

Coincides with classicality in general probabilistic theories, i.e., simplex-embeddability arXiv:1911.10386v2

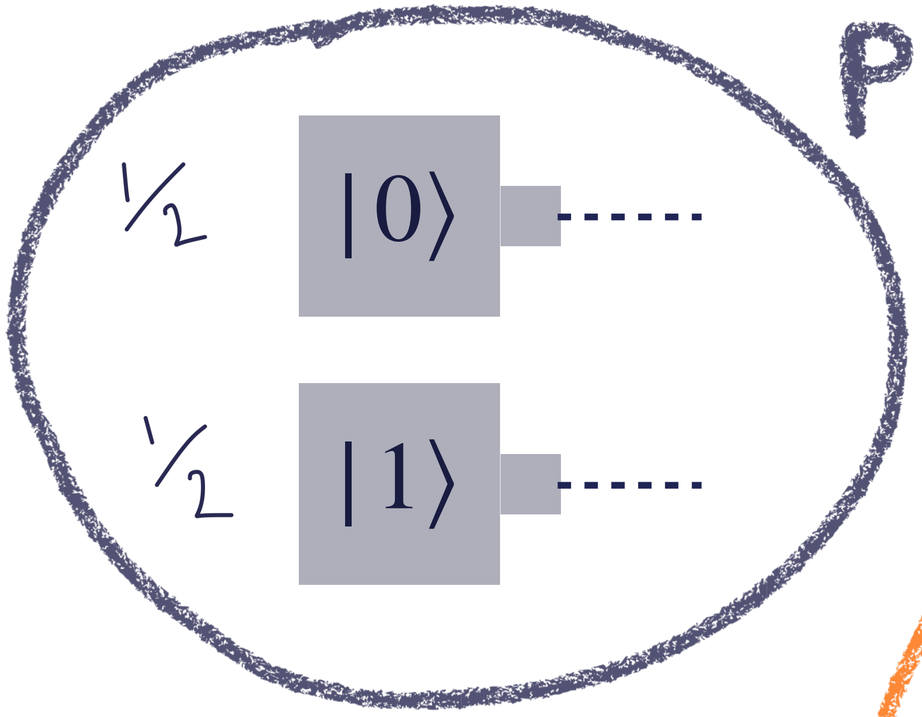


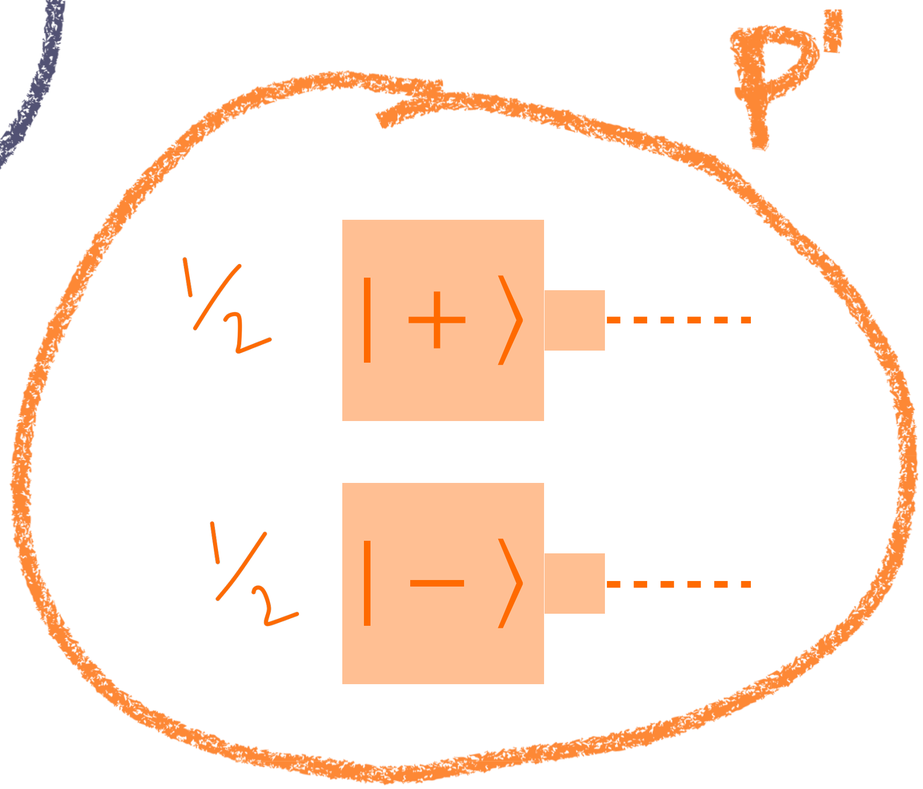


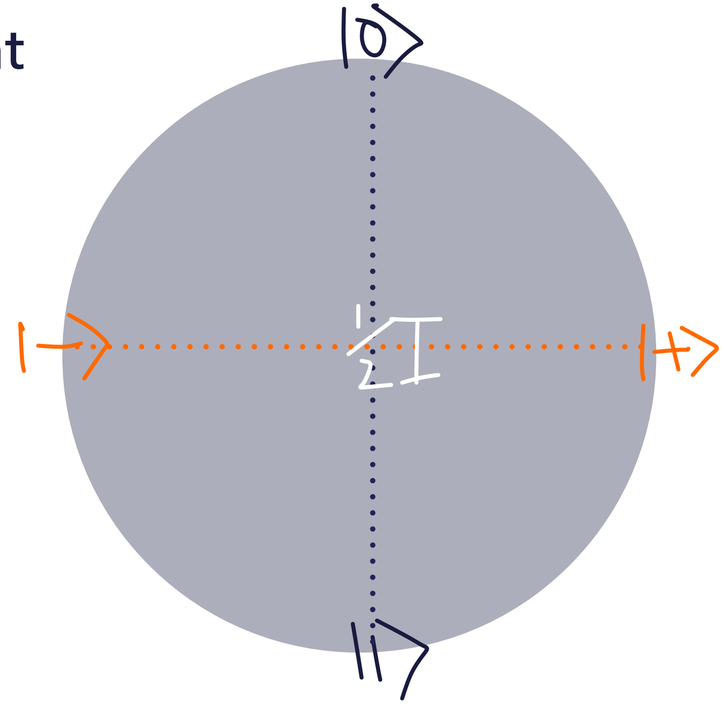


Preparation contexts

Different ways of preparing a system that cannot be distinguished by any measurement







GENERIC OPERATIONAL THEORY:

$\mathscr{P}$ preparations $\quad$ $\mathscr{M}$ measurements $\quad$ $\mathscr{E}$ effects

$P, P' \in \mathcal{P}$ are equivalent: $\quad P \simeq P'$

$$\mathrm{Prob}(k|P, M) = \mathrm{Prob}(k|P', M) \qquad \text{for all } [k|M] \in \mathcal{E}$$

$[k|M], [k'|M'] \in \mathcal{E}$ are equivalent: $\quad [k|M] \simeq [k'|M']$

$$\mathrm{Prob}(k|P, M) = \mathrm{Prob}(k'|P, M') \qquad \text{for all } P \in \mathcal{P}$$

NONCONTEXTUAL ONTOLOGICAL MODEL

Λ, Σ

ontic state space

$\mu_P : \Sigma \to [0,1]$

epistemic state

$\xi_M(k|\lambda)$

response function

$$\mathrm{Prob}(k|P,M) = \int_\Lambda \xi_M(k|\lambda)\mu_P(\lambda)d\lambda$$

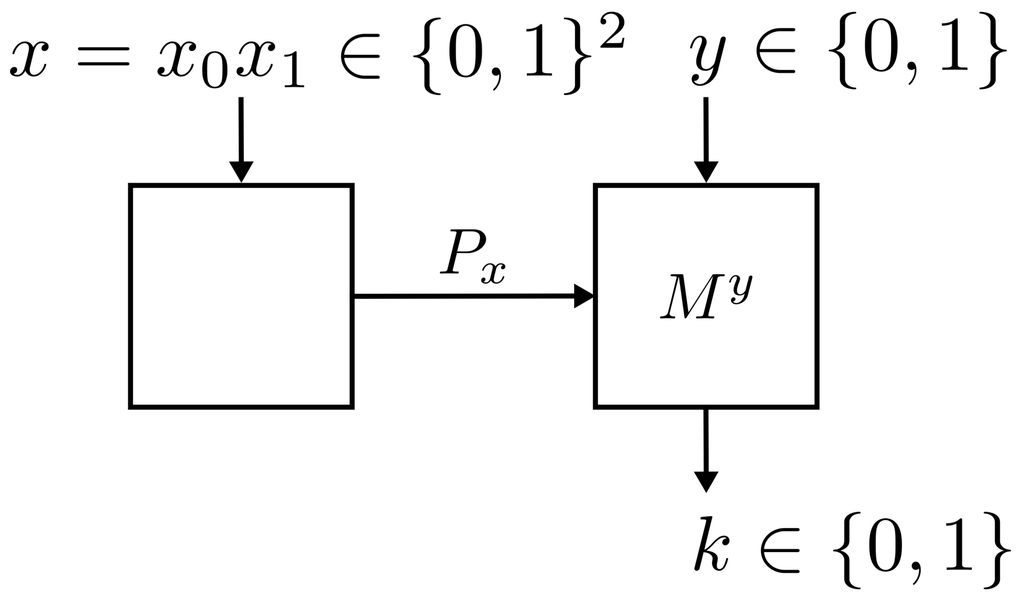
ontological

noncontextual

$$P \simeq P' \quad \Longrightarrow \quad \mu_P = \mu_{P'}$$

$$[k|M] \simeq [k'|M'] \quad \Longrightarrow \quad \xi_M(k|\cdot) = \xi_{M'}(k'|\cdot)$$

RANDOM ACCESS CODES



Win when:

$$k = x_y$$

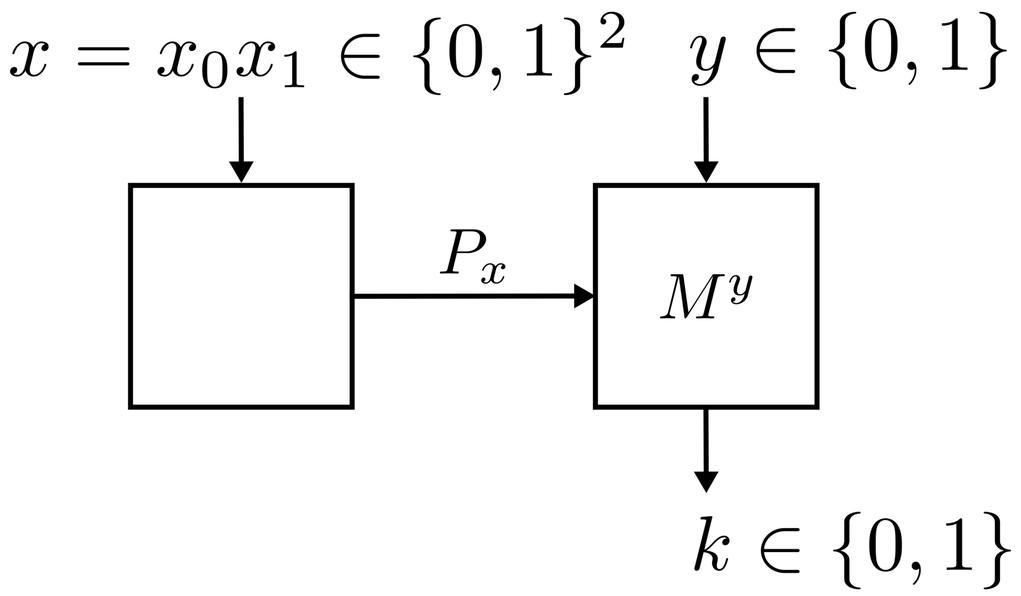
Average success probability:

$$S^{rac}(\mathbf{p}) = \frac{1}{8}\sum_{x,y} p(x_y|x,y)$$

$$S^{rac}_{\mathcal{NC}} = \frac{3}{4}$$

arXiv:1506.04178
arXiv:1710.08434

RANDOM ACCESS CODES



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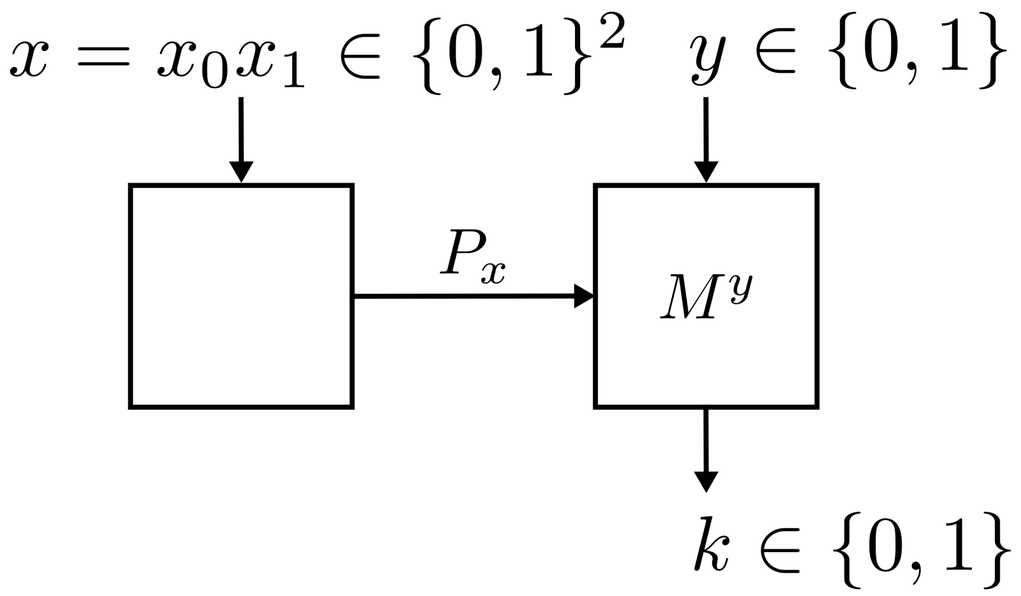
PARITY-OBLIVIOUS
RANDOM ACCESS CODES



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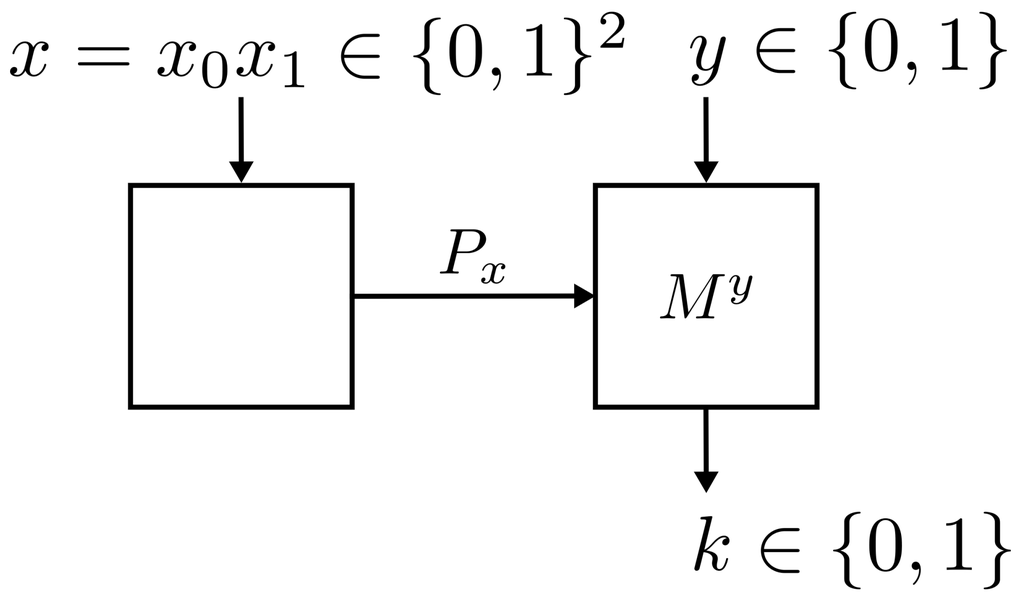

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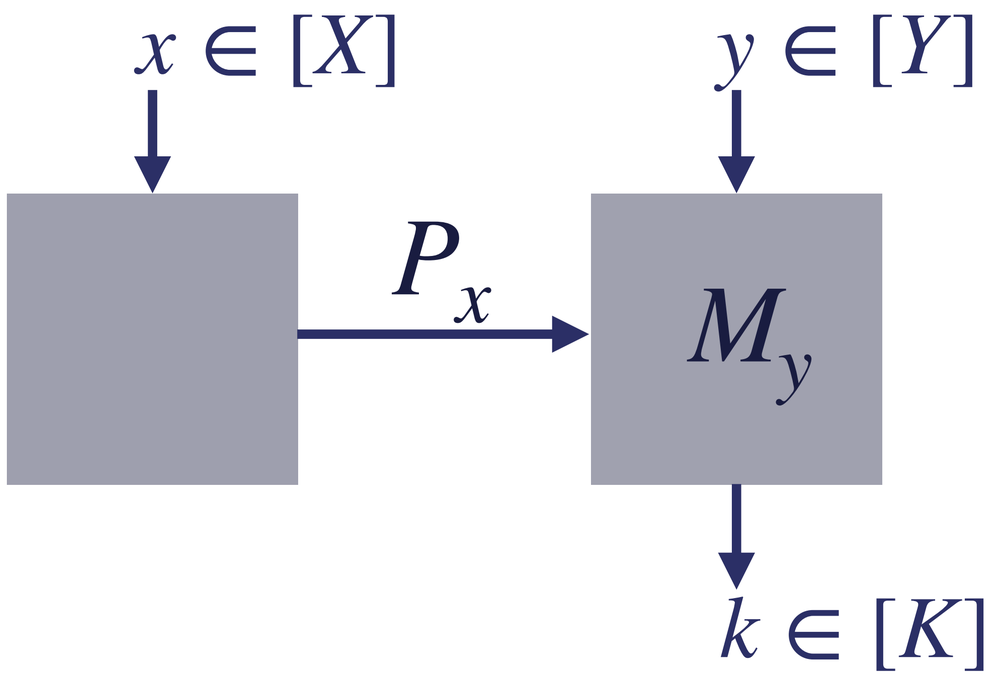
$$S^{rac}_{\mathcal{NC}} = \frac{3}{4} < S^{rac}_{\mathcal{Q}} = \frac{1}{2}(1 + \frac{1}{\sqrt{2}})$$

arXiv:1506.04178
arXiv:1710.08434

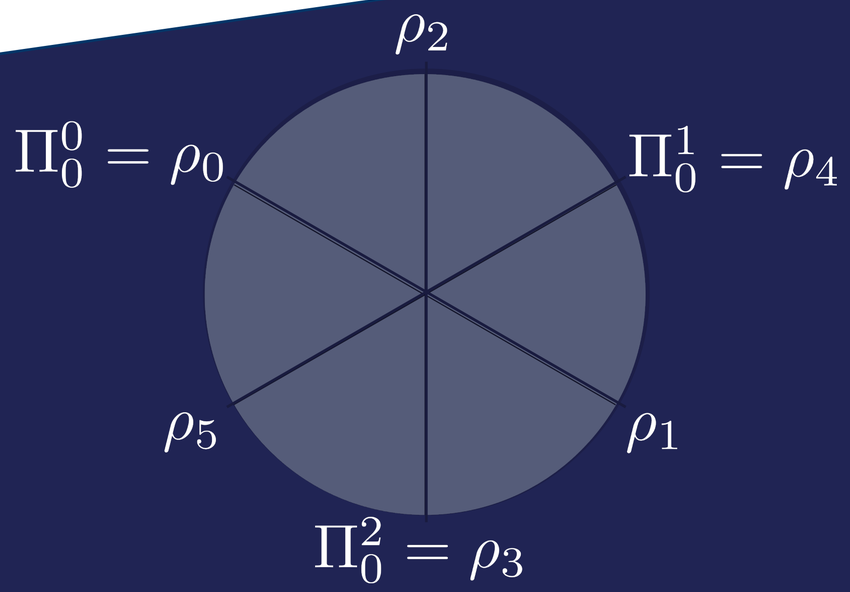
CONTEXTUALITY SCENARIO



$$T = (X, Y, K, \mathcal{OE}_P, \mathcal{OE}_M)$$

$$p(k|x,y) \qquad \mathbf{p} \in \mathbb{R}^{XYK}$$

EXAMPLE



$$\frac{1}{2}(P_0 + P_1) \simeq \frac{1}{2}(P_2 + P_3) \simeq \frac{1}{2}(P_4 + P_5)$$

$$\frac{1}{3}([0|M_0] + [0|M_1] + [0|M_2]) \simeq \frac{1}{3}([1|M_0] + [1|M_1] + [1|M_2])$$

CONTEXTUALITY SCENARIOS THAT "MAP TO" BELL SCENARIOS

Only preparation equivalences

$$\frac{1}{2}(P_1 + P_2) \simeq \frac{1}{2}(P_3 + P_4) \simeq \frac{1}{2}(P_5 + P_6)$$

$$\frac{1}{3}([0\,|\,M_1] + [0\,|\,M_2] + [0\,|\,M_3]) \simeq \frac{1}{3}([1\,|\,M_1] + [1\,|\,M_2] + [1\,|\,M_3])$$

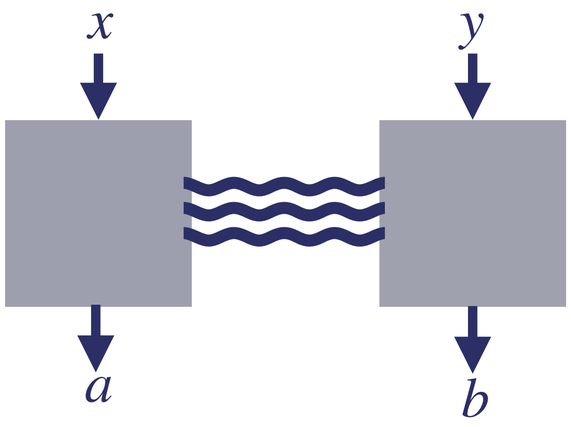
AND

Multiple decompositions of one preparation

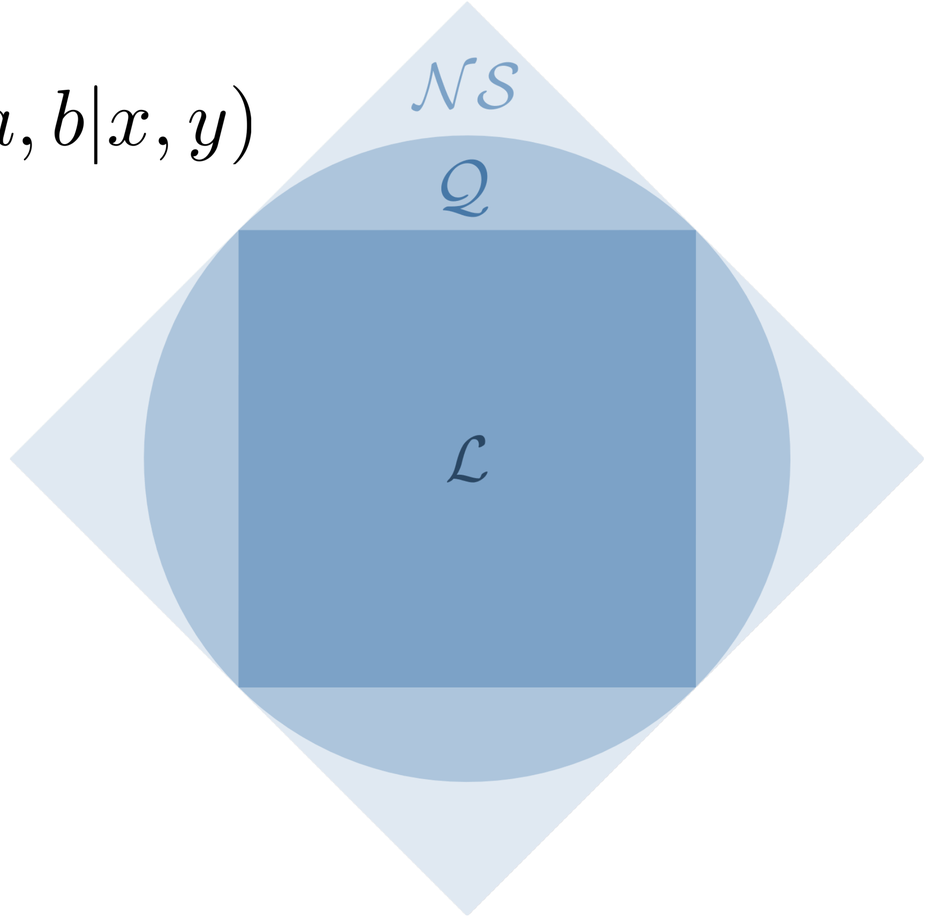
$$\frac{1}{2}(P_1 + P_2) \simeq \frac{1}{2}(P_3 + P_4) \simeq \frac{1}{2}(P_5 + P_6)$$

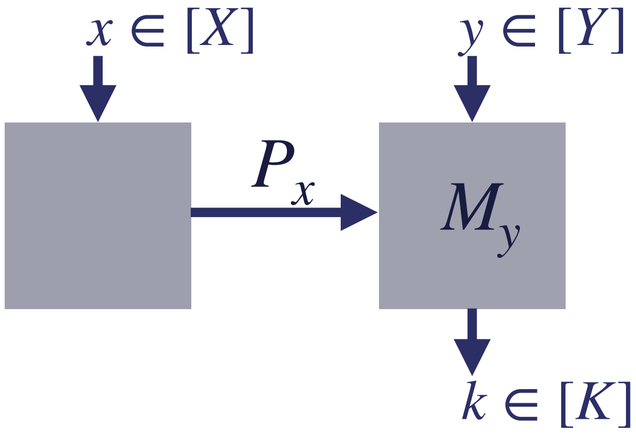
$$\frac{1}{2}(P_1 + P_2) \simeq \frac{1}{2}(P_3 + P_4) \text{ AND } \frac{1}{2}(P_5 + P_6) \simeq \frac{1}{2}(P_7 + P_8)$$

Liang, Spekkens, Wiseman arXiv:1010.1273

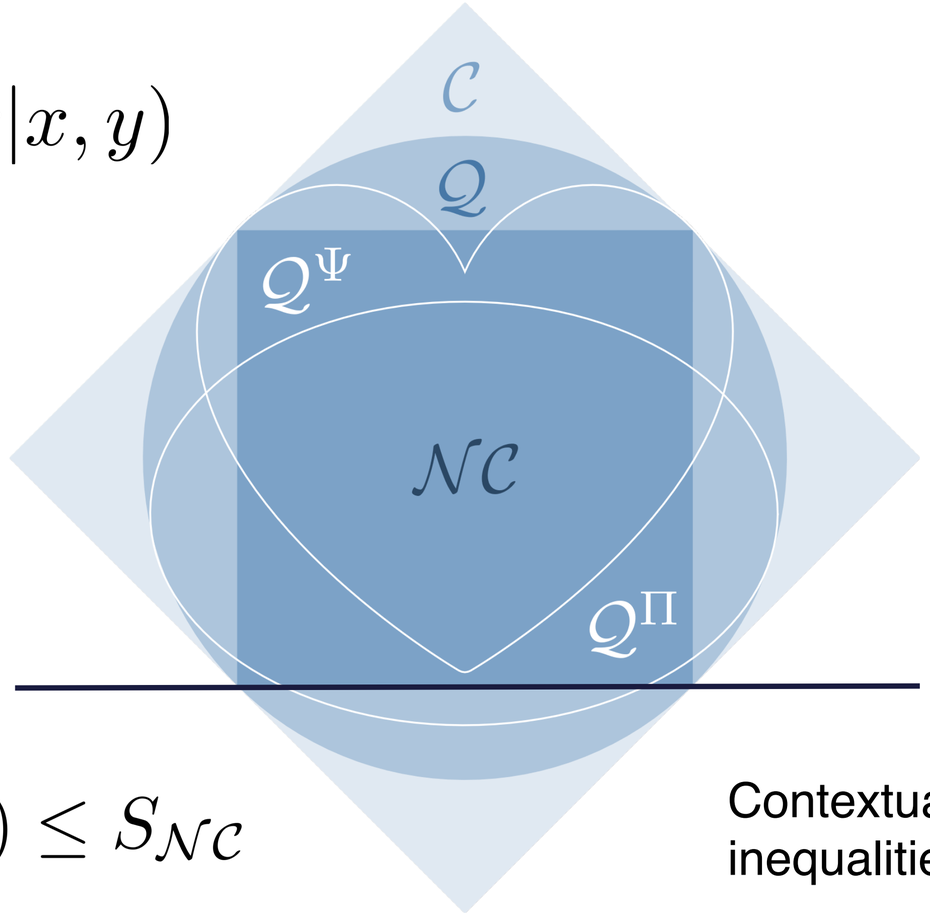


$p(a,b|x,y)$





$p(k|x,y)$



$S(\mathbf{p}) \leq S_{\mathcal{NC}}$

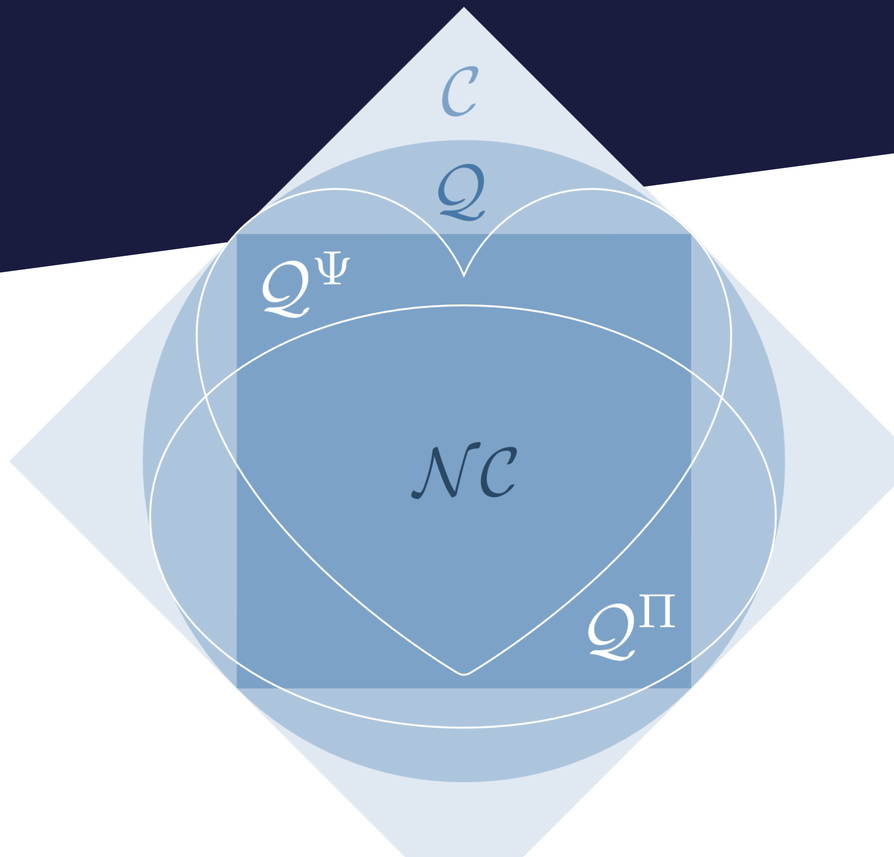
Contextuality inequalities

Chaturvedi, Farkas, VJW
arXiv:2010.05853

Tavakoli, Zambrini Cruzeiro, Uola, Abbott,
arXiv:2010.04751

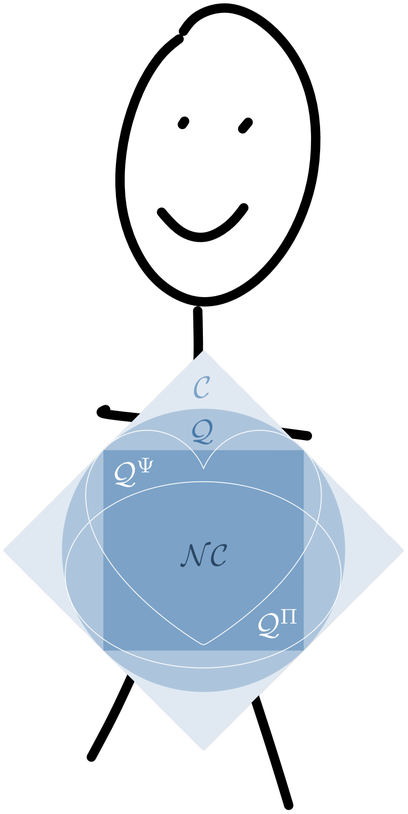
There exist hierarchies of semidefinite programs to approximate $\mathcal{Q}$ from the outside (like NPA for Bell scenarios)

Navascues, Pironio, Acín arXiv:0803.4290



But not much else is known about $\mathcal{Q}$


you
$\mathcal{C}$
$\mathcal{Q}$
$\mathcal{Q}^{\Psi}$
$\mathcal{NC}$
$\mathcal{Q}^{\Pi}$
$\mathcal{NS}$
$\mathcal{Q}$
$\mathcal{L}$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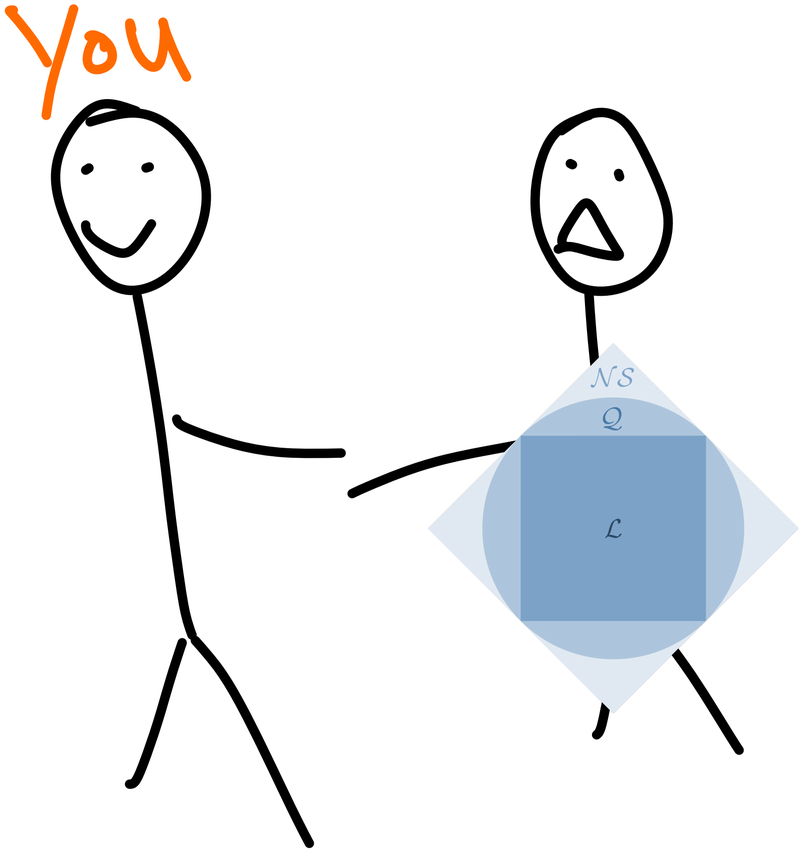
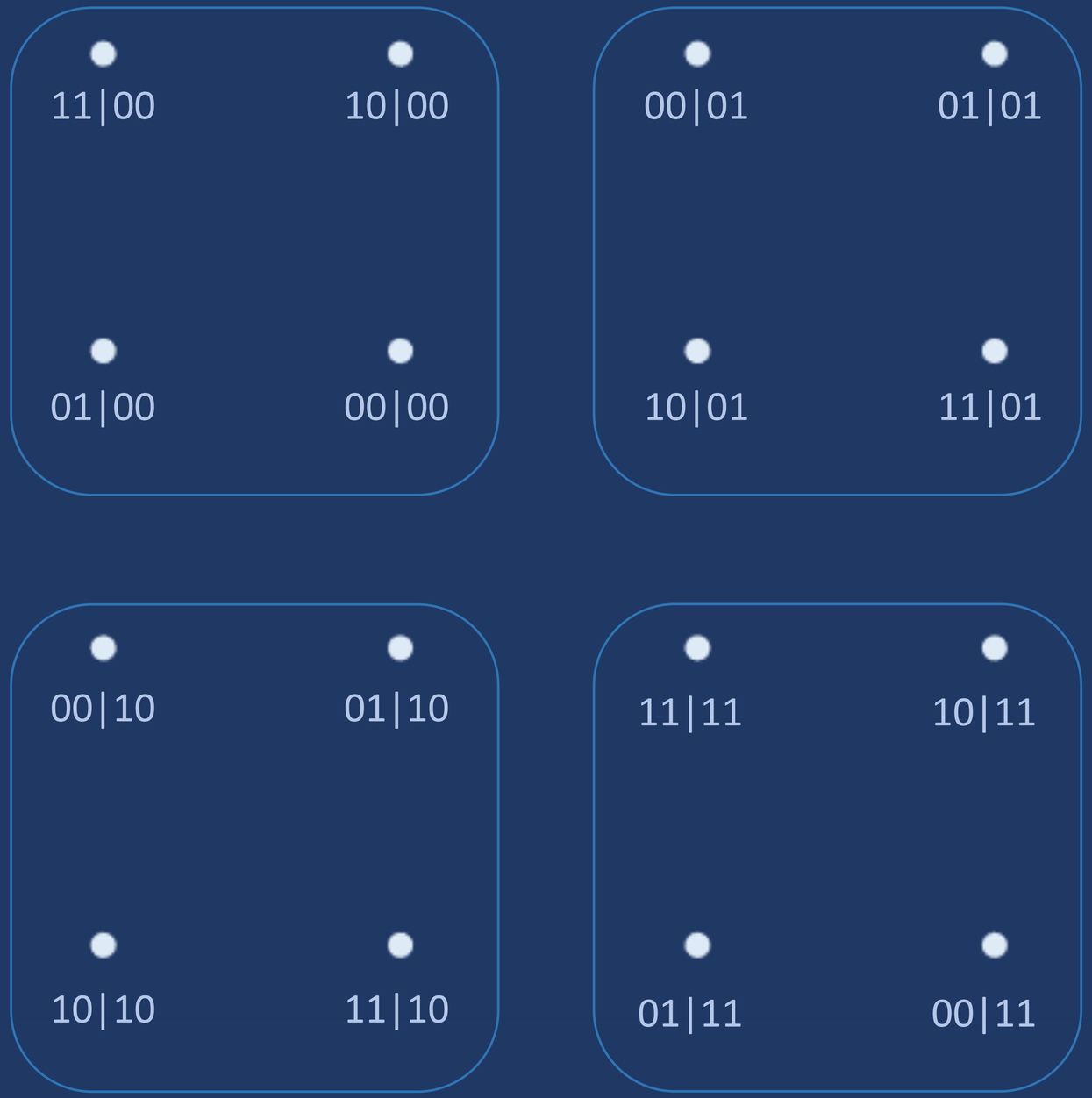

You
$\mathcal{NS}$
$\mathcal{Q}$
$\mathcal{L}$

11|00	10|00	00|01	01|01
01|00	00|00	10|01	11|01
00|10	01|10	11|11	10|11
10|10	11|10	01|11	00|11

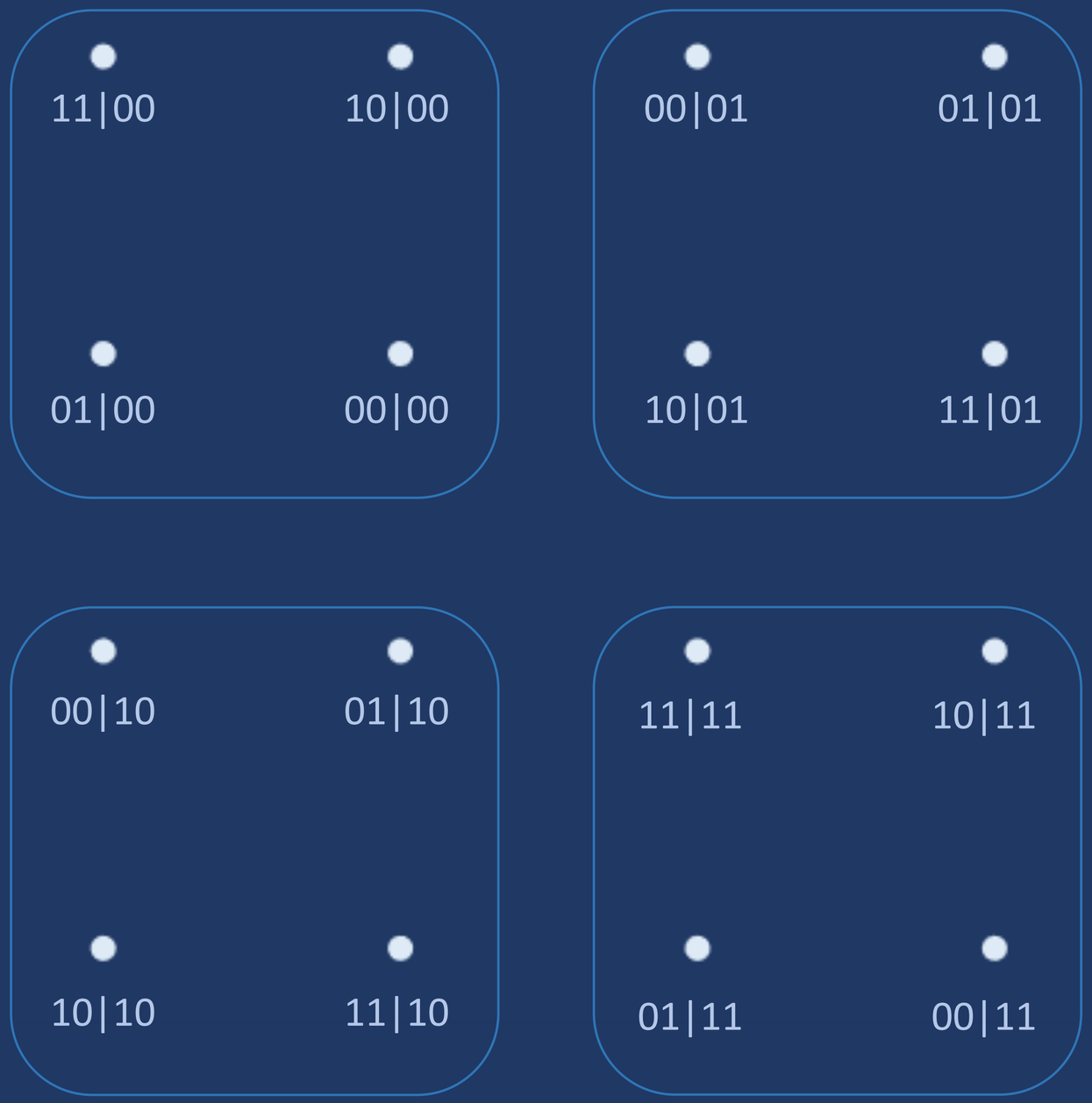
CHSH


11|00
10|00
01|00
00|00
00|01
01|01
10|01
11|01
00|10
01|10
10|10
11|10
11|11
10|11
01|11
00|11

CHSH

No-signalling B→A

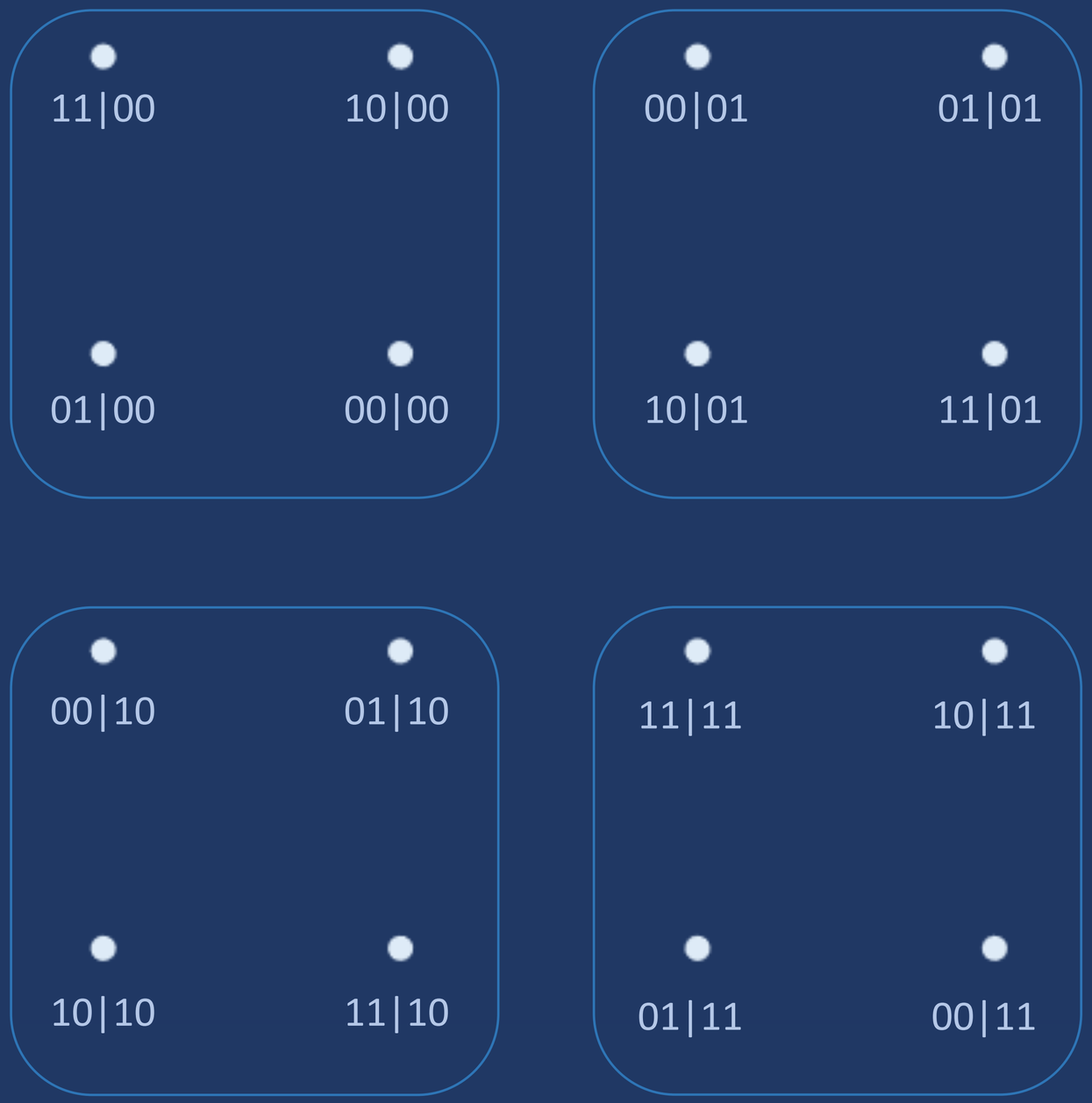
$$p(11|00) + p(10|00)$$
$$= p(10|01) + p(11|01)$$



CHSH

No-signalling B→A

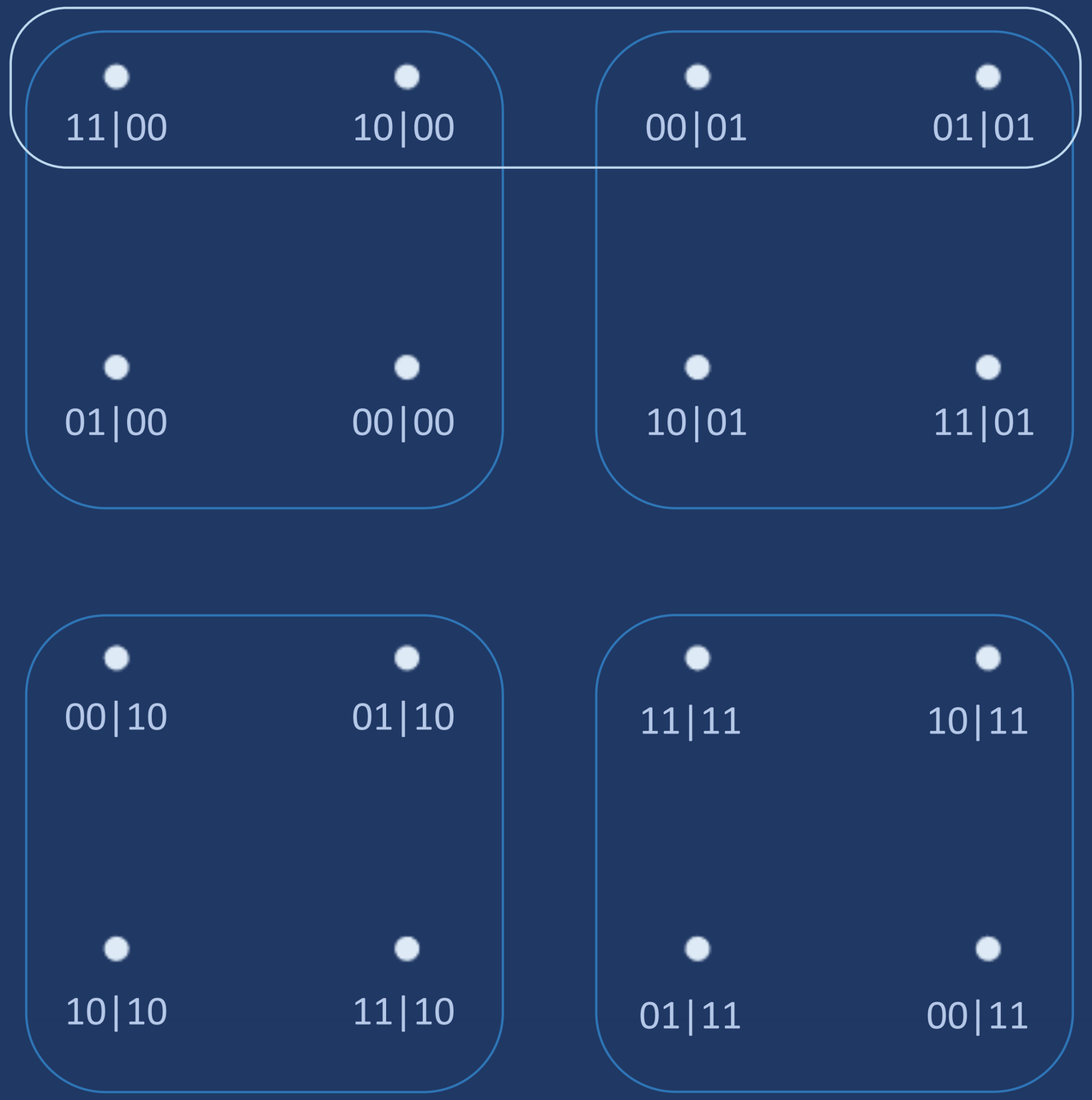
$$
\begin{aligned}
& p(11|00) + p(10|00) \\
&= p(10|01) + p(11|01) \\
&= 1 - p(00|01) - p(01|01)
\end{aligned}
$$



CHSH

No-signalling B→A

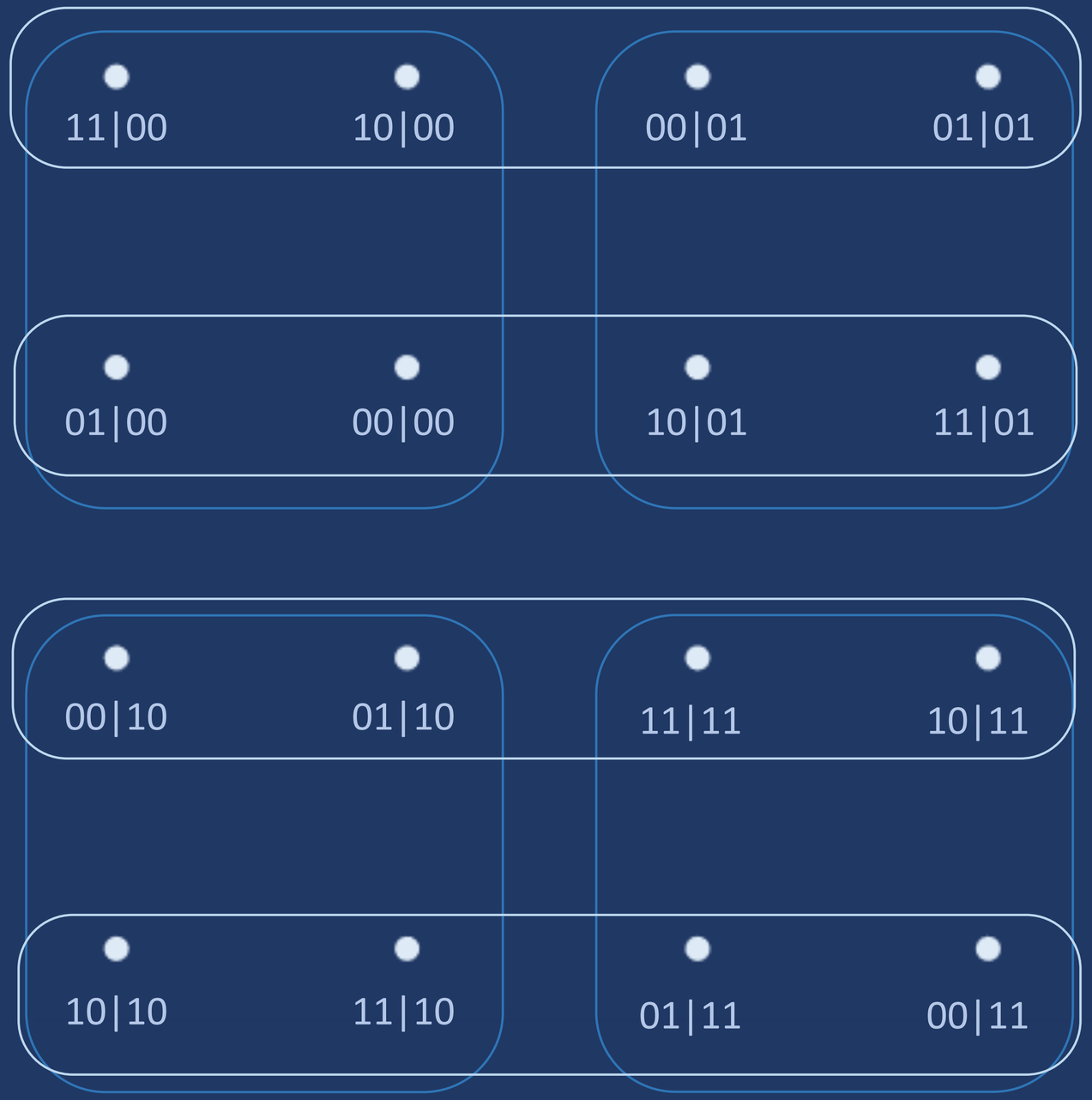
$$
\begin{aligned}
& p(11|00) + p(10|00) \\
&= p(10|01) + p(11|01) \\
&= 1 - p(00|01) - p(01|01)
\end{aligned}
$$



CHSH

No-signalling B→A

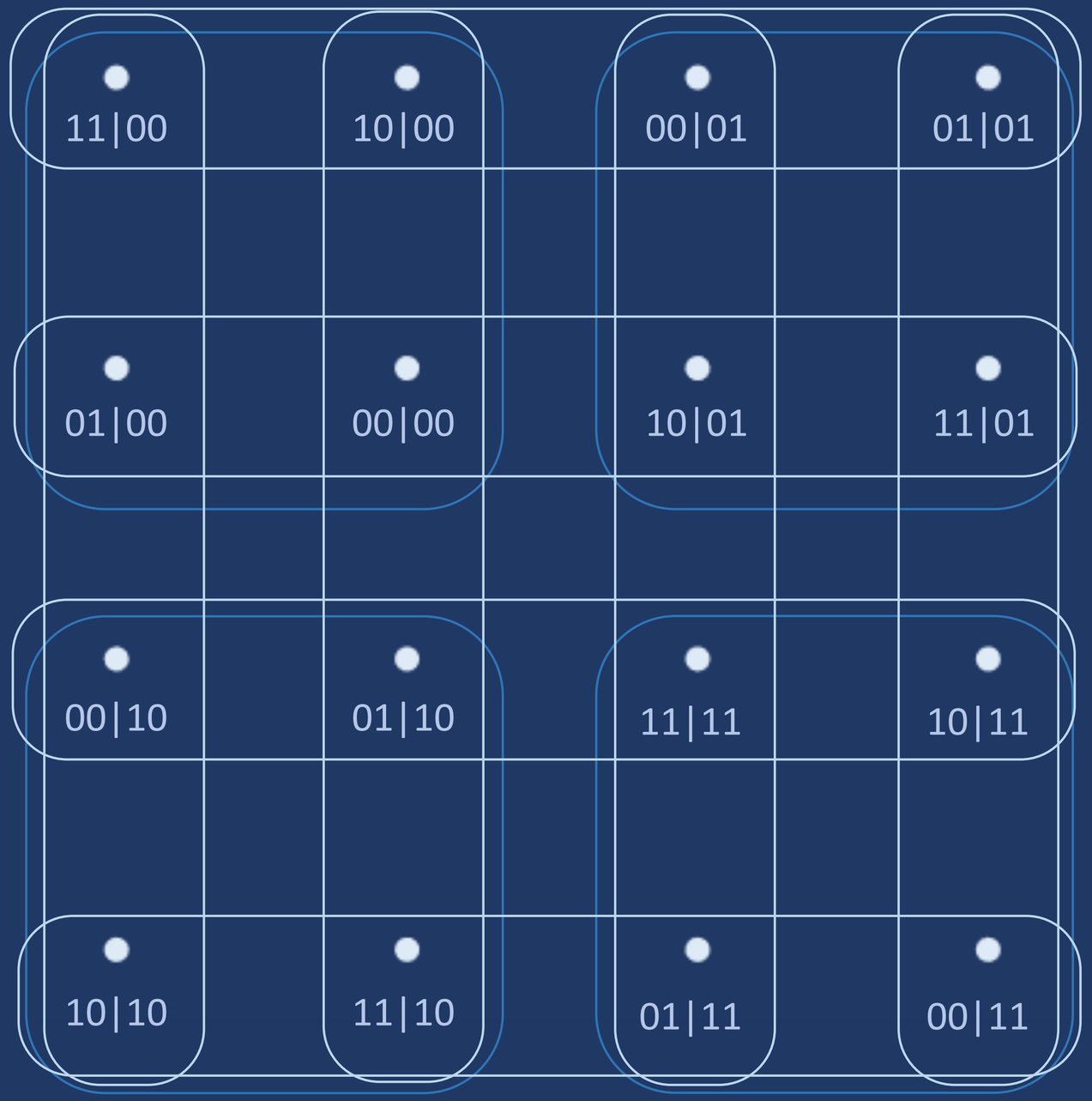
$$
\begin{aligned}
& p(11|00) + p(10|00) \\
&= p(10|01) + p(11|01) \\
&= 1 - p(00|01) - p(01|01)
\end{aligned}
$$



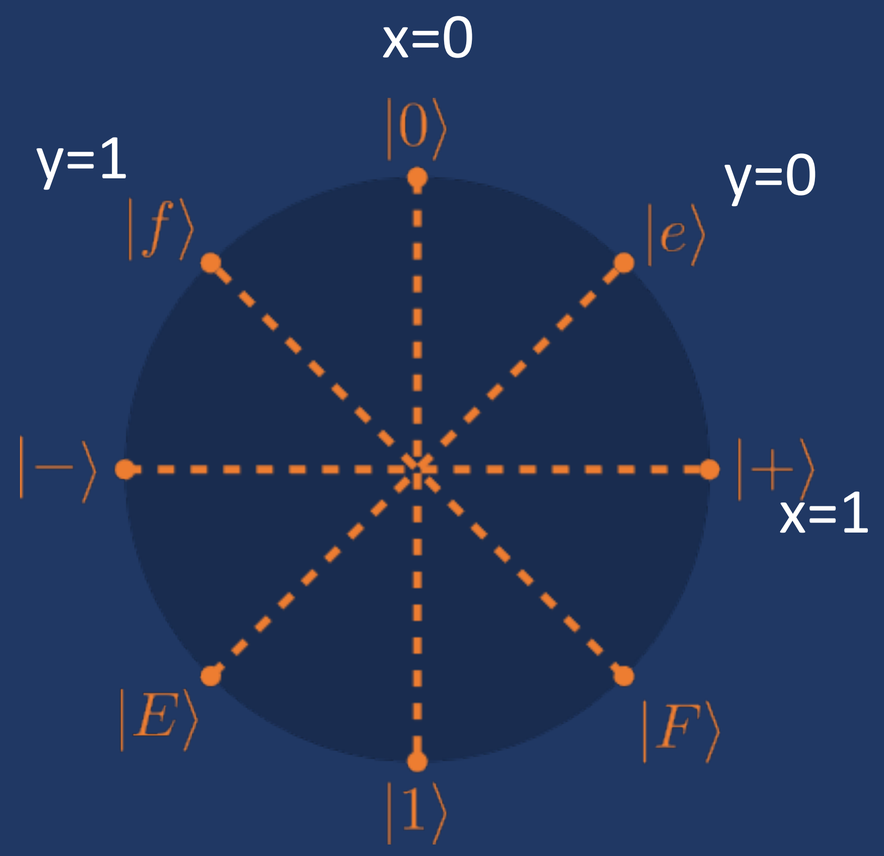
CHSH

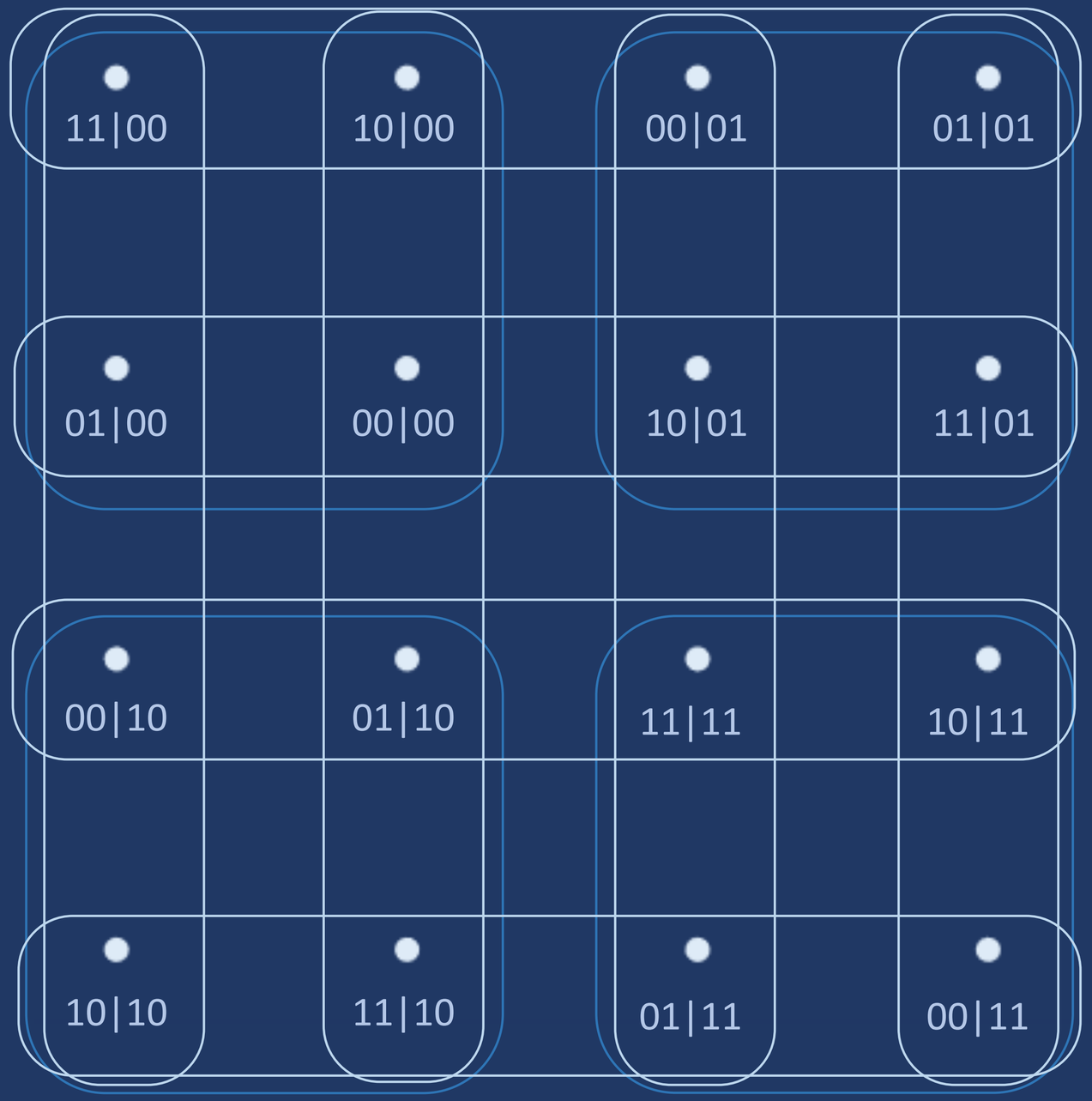
No-signalling B→A

$$p(11|00) + p(10|00)$$
$$= p(10|01) + p(11|01)$$
$$= 1 - p(00|01) - p(01|01)$$

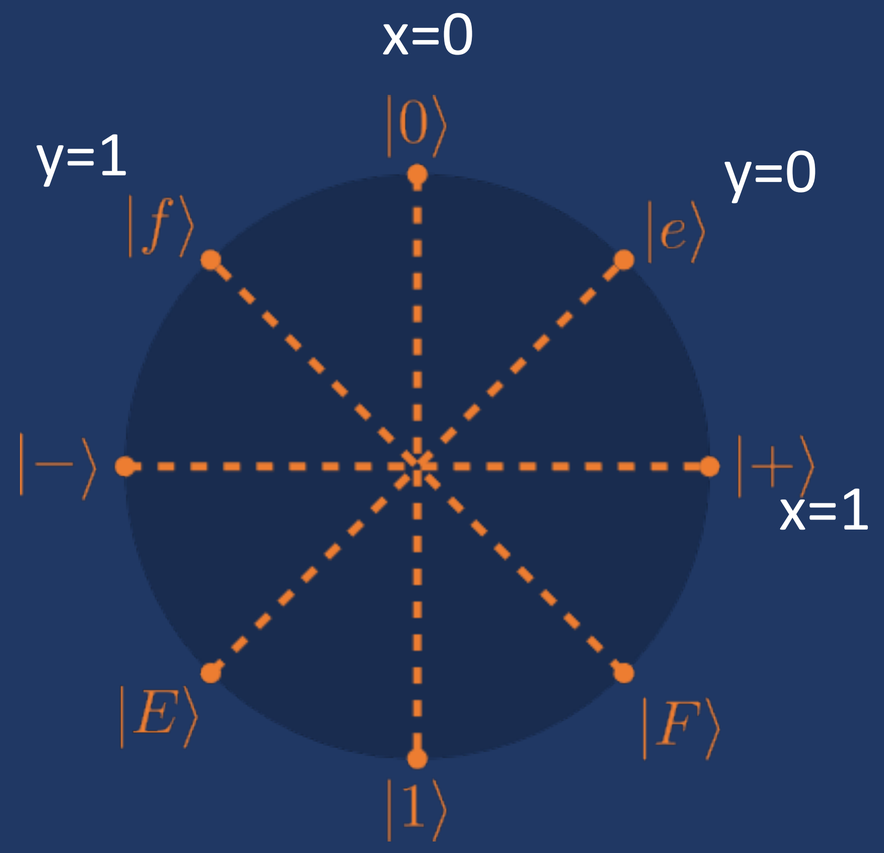


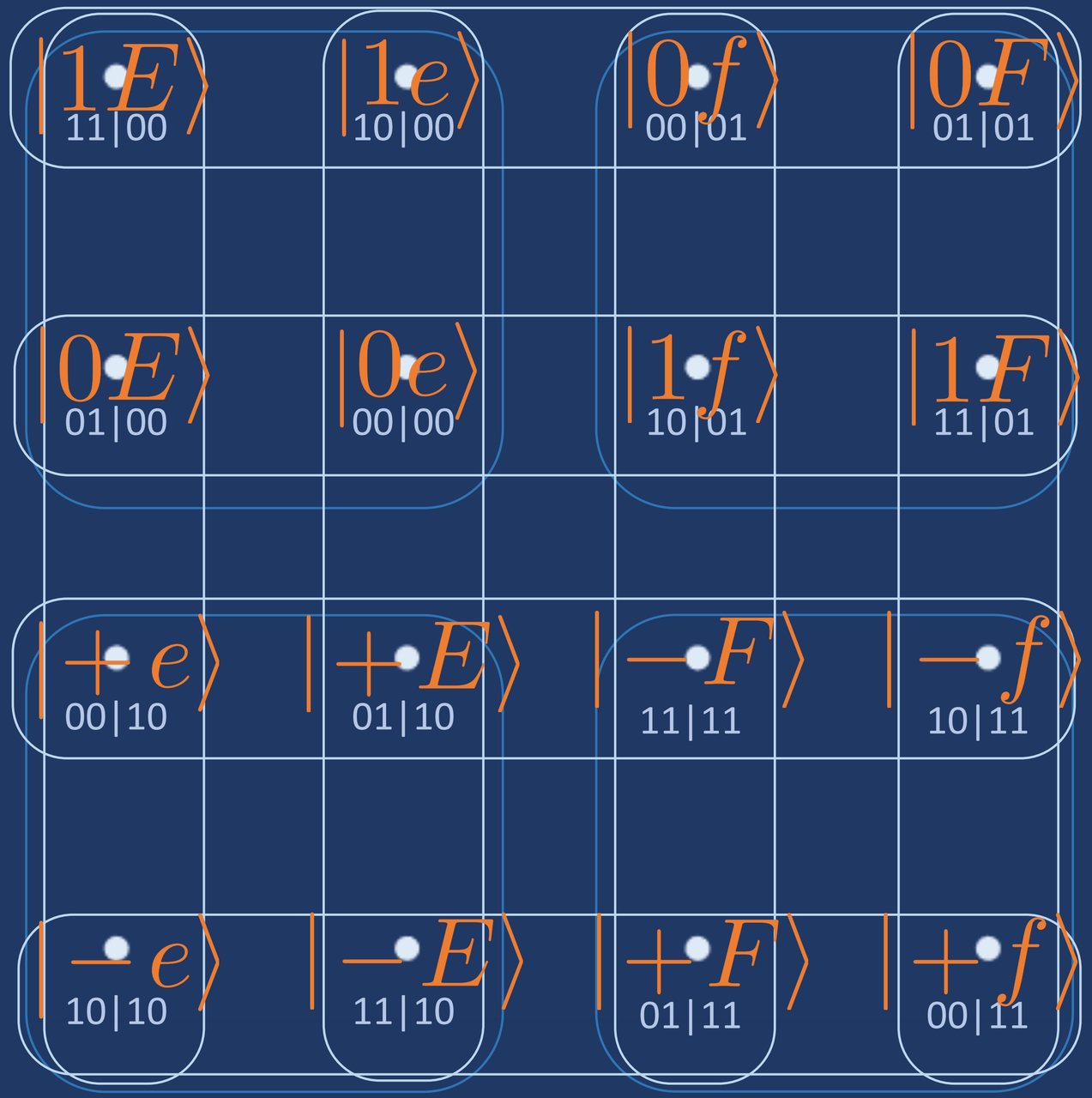
Quantum strategy



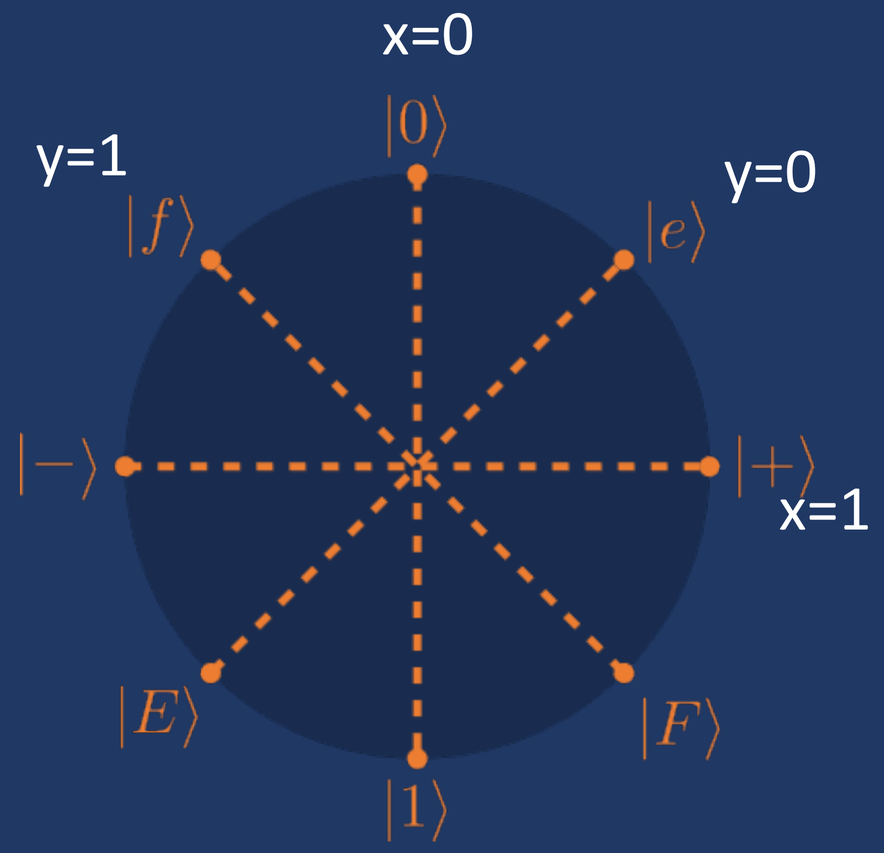


Quantum strategy





Quantum strategy



Local correlations $\cong$ NC statistics