Strong Asymptotics of Planar Orthogonal Polynomials:
Gaussian Weight Perturbed by Point Charges

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Abstract

We consider the planar orthogonal polynomials \( \{ p_n(z) \} \) with respect to the measure supported on the complex plane

\[
e^{-N|z|^2} \prod_{j=1}^{\nu} |z-a_j|^{2c_j} \, dA(z)
\]

where \( dA \) is the Lebesgue measure of the plane, \( N \) is a positive constant, \( \{c_1, \cdots, c_\nu\} \) are nonzero real numbers greater than 1 and \( \{a_1, \cdots, a_\nu\} \subset \mathbb{D} \setminus \{0\} \) are distinct points inside the unit disk. The orthogonal polynomials are related to the interacting Coulomb particles with charge +1 for each, in the presence of extra particles with charge +c_j at a_j. For fixed c_j, these can be considered as small perturbations of the Gaussian weight. When \( \nu = 1 \), in the scaling limit \( n/N = 1 \) and \( n \to \infty \), we obtain strong asymptotics of \( p_n(z) \) via a matrix Riemann–Hilbert problem. From the asymptotic behavior of \( p_n(z) \), we find that, as we vary \( c_1 \), the limiting distribution of zeros behaves discontinuously at \( c_1 = 0 \). We observe that the generalized Szegő curve (a kind of potential theoretic skeleton) also behaves discontinuously at \( c_1 = 0 \). We also derive the strong asymptotics of \( p_n(z) \) for the case of \( \nu > 1 \) by applying the nonlinear steepest descent method on the matrix Riemann-Hilbert problem of size \((\nu + 1) \times (\nu + 1)\). This talk is based on joint work with Seung-Yeop Lee.