Strong Asymptotics of Planar Orthogonal Polynomials: Gaussian Weight Perturbed by Point Charges

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Abstract

We consider the planar orthogonal polynomials $\{p_n(z)\}$ with respect to the measure supported on the complex plane

$$\mathrm{e}^{-N|z|^2} \prod_{j=1}^{\nu} |z-a_j|^{2c_j} \, \mathrm{d}A(z)$$

where $\mathrm{d}A$ is the Lebesgue measure of the plane, N is a positive constant, $\{c_1, \cdots, c_\nu\}$ are nonzero real numbers greater than -1 and $\{a_1, \cdots, a_\nu\} \subset \mathbb{D} \setminus \{0\}$ are distinct points inside the unit disk. The orthogonal polynomials are related to the interacting Coulomb particles with charge $+1$ for each, in the presence of extra particles with charge $+c_j$ at a_j. For fixed c_j, these can be considered as small perturbations of the Gaussian weight. When $\nu = 1$, in the scaling limit $n/N = 1$ and $n \to \infty$, we obtain strong asymptotics of $p_n(z)$ via a matrix Riemann–Hilbert problem. From the asymptotic behavior of $p_n(z)$, we find that, as we vary c_1, the limiting distribution of zeros behaves discontinuously at $c_1 = 0$. We observe that the generalized Szegő curve (a kind of potential theoretic skeleton) also behaves discontinuously at $c_1 = 0$. We also derive the strong asymptotics of $p_n(z)$ for the case of $\nu > 1$ by applying the nonlinear steepest descent method on the matrix Riemann-Hilbert problem of size $(\nu + 1) \times (\nu + 1)$. This talk is based on joint work with Seung-Yeop Lee.