Title: Geometric interpretation of the Lieb–Thirring conjecture

Abstract: Consider the Laplace operator with a negative potential V in n dimensions. Such a system would posses bound states with negative energies $e_1 \leq e_2 \leq \cdots < 0$.

For their proof of stability of matter, Lieb and Thirring proved and used the following upper bound on the sum of the absolute values of the energies raised to a power γ

$$\sum_{j\geq 1} |e_j|^\gamma \leq L_{\gamma,n} \int_{\mathbb{R}^n} V^{\gamma+n/2}\, \mathrm{d}x\,.$$

This inequality is known after its authors as the Lieb–Thirring inequality. The optimal constant $L_{\gamma,1}$ is not known in dimension one for the values of $\gamma \in (1/2, 3/2)$. Lieb and Thirring however conjectured, that the sharp constant $L_{\gamma,1}$ is achieved for potentials which only have one bound state (which *is* known).

I will review a paper discussing the connection between the Lieb–Thirring conjecture in one dimension and isoperimetric problem for ovals on the plane by R. Benguria and M. Loss from 2004. This is only a reformulation of the conjecture, since the corresponding isoperimetric problem is an open question as well. The authors discuss potentials that host two bound states. Afterwards I will discuss the generalisation of the problem to potentials with three bound states.