Title: The long-time behavior of boundary driven quantum systems near the Zeno limit.

Abstract: We investigate bipartite quantum systems on a Hilbert space $\mathcal{H}_{\mathcal{AB}} = \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$ where the dynamics is given by two parts: A coherent evolution generated by a Hamiltonain H acting on $\mathcal{H}_{\mathcal{AB}}$, and a Lindbladian dissipator $\mathcal{D} = \mathcal{D}_{\mathcal{A}} \otimes \mathcal{I}_{\mathcal{B}}$ acting non-trivially only on operators on $\mathcal{H}_{\mathcal{A}}$. Define $\mathcal{L}_\gamma$ by $\mathcal{L}_\gamma \rho := -i[H, \rho] + \gamma \mathcal{D}\rho$. The evolution equation we study is

$$\dot{\rho}(t) = \mathcal{L}_\gamma \rho(t) \ , \qquad (*)$$

where γ is a constant taken to be large. The limit $\gamma \to \infty$ is known as the Zeno limit. We assume that $\mathcal{D}_A$ is ergodic with a unique steady state π_A and has a spectral gap. Then $\mathcal{P} = \lim_{t\to\infty} e^{t\mathcal{D}}$ is the projection onto the nullspace of $\mathcal{D}$, which consists of all operators on $\mathcal{H}_{AB}$ of the form $\pi_A \otimes X$ where X is any operator on $\mathcal{H}_B$. It is known that near the Zeno limit, after a short time t_0 of order γ^{-1}, solutions $\rho(t)$ of $(*)$ satisfy $\|\rho(t) - \mathcal{P}\rho(t)\|_1 = \mathcal{O}(\gamma^{-1})$, uniformly in $t \geq t_0$, where the norm is the trace norm. Define $\mu(t)$ by $\mathcal{P}\rho(t) = \pi_A \otimes \mu(t)$. We study the evolution of $\mu(t)$ and derive an effective equation for this that is valid, in the Zeno limit, on arbitrary time intervals, and we apply this to the study of stationary states of $(*)$. There is a useful analogy with the theory of hydrodynamic limits which is exploited in this work. One may think of $(*)$ as an analog of the Boltzmann equation with γ^{-1} corresponding to the Knudsen number, which one takes to zero in the hydrodynamic limit. One may think of $\mu(t)$ as corresponding to the hydrodynamic moments. Then making appropriate rescalings of space and time involving the Knudsen number, one obtains the Euler equations or the Navier-Stokes equations, depending on the rescaling. Note that these hydrodynamic equations do not involve the Knudsen number, and neither does our effective equation for $\mu(t)$, setting it apart from previously derived approximate equations for $\mu(t)$ that are valid on shorter time scales, and that are less well-adapted to the study of stationary states. This is joint work with David Huse and Joel Lebowitz.