

The many facets of trisqueezing

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Conventional (bi)squeezing is one of the cornerstones of quantum optics. Higher-order generalizations of the form $(a^\dagger)^3 + a^3$ (‘trisqueezing’) were popularized by Braunstein in the 80s as a natural extension of the squeezing paradigm into the multi-photon regime. These would allow going beyond Gaussian operations toward genuinely nonlinear, non-Gaussian resources. Ever since then, there has been a longstanding debate on the well-definedness and uniqueness of the dynamics generated by trisqueezing, as well as recent experimental realizations in ion traps and superconducting systems.

In this talk we show that additional information about the physical system in the form of boundary conditions for high photon numbers is needed in order to define unique trisqueezed states [1]. These boundary conditions can, for example, be induced by additional free field contributions like Kerr terms $\varepsilon(a^\dagger a)^3$. Such multi-photon interactions penalize high photon numbers and thus effectively cut off the system at high energies, hence implementing the aforementioned boundary conditions. Of particular interest is the case where these additional multi-photon interactions are small in the sense that $\varepsilon \ll 1$, and the trisqueezing Hamiltonian is expected to be a good approximation of the system at hand. In this case, the boundary conditions, and therefore the trisqueezing dynamics are extremely sensitive against small changes in ε : we show that infinitely many possible realizations (‘facets’; technically: self-adjoint extensions) of trisqueezing can be achieved by small variations of the Kerr contributions [2]. Two particularly relevant facets, H_{even} and H_{odd} , occur naturally in numerics through even/odd sized Fock-space truncations of trisqueezing [3]. Since the different facets of trisqueezing generate very different states (see Figure below), this shows that these small variations in ε matter physically. This begs the question; which facets are selected in the actual experiments?

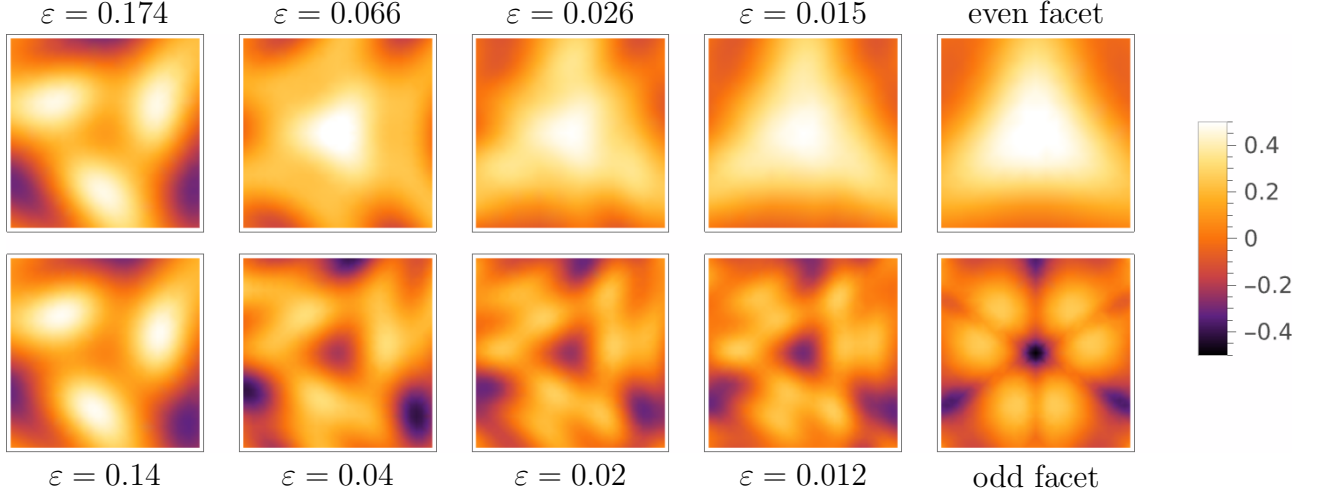


Figure 1: Wigner functions of trisqueezed states $\exp \{ -i ((a^\dagger)^3 + a^3 + \varepsilon(a^\dagger a)^3) \} |0\rangle$ for different perturbation strength ε . Horizontally shown are two sequences $\varepsilon \rightarrow 0$ *provably* converging to the evolved state of two different facets $\exp \{ -iH_{\text{even}} \} |0\rangle$ and $\exp \{ -iH_{\text{odd}} \} |0\rangle$ of $(a^\dagger)^3 + a^3$. When the figure is read vertically it shows the high sensitivity of the state with respect to ε .

- [1] F. Fischer, D. Burgarth, D. Lonigro, *Self-adjoint realizations of higher-order squeezing operators*, J. Phys. A: Math. Theor. **59** 255203 (2026).
- [2] F. Fischer, D. Burgarth, D. Lonigro, *Essentially singular limits of Jacobi operators and applications to higher-order squeezing*, arXiv:2605.21355 (2026).
- [3] S. Ashhab, F. Fischer, D. Lonigro, D. Braak, D. Burgarth, *Finite-dimensional approximations of generalized squeezing*, Phys. Rev. A **113** 013703 (2026).